WINTER– 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22206**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	If $f(x)=x^2-x+1$, find $f(0)+f(3)$	**02**
	Ans	$f(0)=(0)^2-0+1=1$	½
		$f(3)=(3)^2-3+1=7$	1
		$\therefore f(0)+f(3)=1+7$	
		$=8$	½
		--	
	b)	Show that $f(x)=\dfrac{a^x+a^{-x}}{2}$ is an even function	**02**
	Ans	$f(x)=\dfrac{a^x+a^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{a^{-x}+a^{-(-x)}}{2}$	½
		$\therefore f(-x)=\dfrac{a^{-x}+a^{x}}{2}=\dfrac{a^x+a^{-x}}{2}$	½
		$\therefore f(-x)=f(x)$	½
		$\therefore$ function is even	½
		--	
	c)	Find $\dfrac{dy}{dx}$, if $y=x^5+5^x+e^x+\log_2 x$	**02**
	Ans	$y=x^5+5^x+e^x+\log_2 x$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	$\therefore y = x^5 + 5^x + e^x + \dfrac{\log x}{\log 2}$ $\therefore \dfrac{dy}{dx} = 5x^4 + 5^x \log 5 + e^x + \dfrac{1}{\log 2}\dfrac{1}{x}$ $\therefore \dfrac{dy}{dx} = 5x^4 + 5^x \log 5 + e^x + \dfrac{1}{x \log 2}$	2
	d)	Evaluate $\int \dfrac{1}{1+\cos 2x}\, dx$	**02**
	Ans	$\int \dfrac{1}{1+\cos 2x}\, dx$ $= \int \dfrac{1}{2\cos^2 x}\, dx\, dx$ $= \dfrac{1}{2} \int \sec^2 x\, dx$ $= \dfrac{1}{2}\tan x + c$	 ½ ½ 1
	e)	Evaluate $\int x.e^x dx$	**02**
	Ans	$\int x.e^x dx$ $= x\left(\int e^x dx\right) - \int\left(\int e^x dx \dfrac{d}{dx}(x)\right)dx$ $= xe^x - \int e^x.1\, dx$ $= xe^x - \int e^x\, dx$ $= xe^x - e^x + c$	 ½ ½ ½ ½
	f)	Find area bounded by the curve $y = x^3$, $x-$axis and the ordinate $x = 1$ to $x = 3$	**02**
	Ans	Area $A = \int_a^b y\, dx$ $= \int_1^3 x^3 dx$ $= \left[\dfrac{x^4}{4}\right]_1^3$	 ½ ½

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f)	$=\left(\frac{3^4}{4}-\frac{1^4}{4}\right)$	½
		$=20$	½
		--	
	g)	If a fair coin is tossed three times, then find the probability of getting exactly two heads.	02
	Ans	$S=\{HHH,HHT,THH,HTH,HTT,THT,TTH,TTT\}$	
		$\therefore n(S)=8$	½
		$A=\{HHT,THH,HTH\}$	
		$\therefore n(A)=3$	½
		$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{3}{8}=0.375$	1
		--	
2.		**Attempt any THREE of the following:**	12
	a)	Find $\frac{dy}{dx}$ if , $e^x+e^y=e^{x+y}$	04
	Ans	$e^x+e^y=e^{x+y}$	
		$\therefore e^x+e^y\frac{dy}{dx}=e^{x+y}\left(1+\frac{dy}{dx}\right)$	2
		$\therefore e^x+e^y\frac{dy}{dx}=e^{x+y}+e^{x+y}\frac{dy}{dx}$	
		$\therefore e^y\frac{dy}{dx}-e^{x+y}\frac{dy}{dx}=e^{x+y}-e^x$	
		$\therefore\left(e^y-e^{x+y}\right)\frac{dy}{dx}=e^{x+y}-e^x$	1
		$\therefore\frac{dy}{dx}=\frac{e^{x+y}-e^x}{e^y-e^{x+y}}$	1
		$\therefore\frac{dy}{dx}=\frac{e^x\left(e^y-1\right)}{e^y\left(1-e^x\right)}$	
		--	
	b)	If $x=a\cos^3\theta$, $y=b\sin^3\theta$. Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$	04
	Ans	$x=a\cos^3\theta$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	$\therefore \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$	1
		$y = b\sin^3\theta$	
		$\therefore \frac{dy}{d\theta} = 3b\sin^2\theta\cos\theta$	1
		$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3b\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	1
		$\therefore \frac{dy}{dx} = -\frac{b\sin\theta}{a\cos\theta}$	
		$\therefore \frac{dy}{dx} = -\frac{b}{a}\tan\theta$	½
		at $\theta = \frac{\pi}{3}$	
		$\therefore \frac{dy}{dx} = -\frac{b}{a}\tan\frac{\pi}{3}$	
		$\therefore \frac{dy}{dx} = -\frac{\sqrt{3}\text{ b}}{a}$	½
	c)	A telegraph wire hangs in the form of a curve $y = a.\log\left[\sec\left(\frac{x}{a}\right)\right]$ where a is constant. Show that, radius of curvature at any point is $a.\sec\left(\frac{x}{a}\right)$	**04**
	Ans	$y = a.\log\sec\left(\frac{x}{a}\right)$	
		$\therefore \frac{dy}{dx} = a\frac{1}{\sec\left(\frac{x}{a}\right)}\sec\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right)\frac{1}{a}$	1
		$\therefore \frac{dy}{dx} = \tan\left(\frac{x}{a}\right)$	½
		$\therefore \frac{d^2y}{dx^2} = \sec^2\left(\frac{x}{a}\right)\frac{1}{a}$	½

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2.	c)	$\therefore$ Radius of curvature $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$	
		$= \dfrac{\left[1+\tan^2\left(\dfrac{x}{a}\right)\right]^{3/2}}{\sec^2\left(\dfrac{x}{a}\right)\dfrac{1}{a}}$	½
		$= \dfrac{a\left[\sec^2\left(\dfrac{x}{a}\right)\right]^{3/2}}{\sec^2\left(\dfrac{x}{a}\right)}$	½
		$= \dfrac{a\sec^3\left(\dfrac{x}{a}\right)}{\sec^2\left(\dfrac{x}{a}\right)}$	½
		$\therefore$ Radius of curvature $\rho = a\sec\left(\dfrac{x}{a}\right)$	½
		--	
	d)	A beam is supported at the two ends and is uniformly loaded. The bending moment M at a distance x from the end is given by $M = \dfrac{Wl}{2}\times x - \dfrac{W}{2}\times x^2$. Find the point at which M is maximum.	**04**
	Ans	$M = \dfrac{Wl}{2}\times x - \dfrac{W}{2}\times x^2$	
		$\therefore \dfrac{dM}{dx} = \dfrac{Wl}{2} - Wx$	1
		$\therefore \dfrac{d^2M}{dx^2} = -W$	1
		Consider $\dfrac{dM}{dx} = 0$	
		$\therefore \dfrac{Wl}{2} - Wx = 0$	
		$\therefore \dfrac{Wl}{2} = Wx$	
		$\therefore x = \dfrac{l}{2}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	at $x=\frac{l}{2}$ $\therefore \frac{d^2M}{dx^2}=-W<0$ $\therefore$ the point $x=\frac{l}{2}$, M is maximum.	 1
		--	
3.		**Attempt any <u>THREE</u> of the following:**	**12**
	a)	Find the equation of tangent and normal to the curve $y=x^2$ at point $\left(-1,1\right)$	**04**
	Ans	$y=x^2$ $\therefore \frac{dy}{dx}=2x$ at $\left(-1,1\right)$ $\therefore \frac{dy}{dx}=2\left(-1\right)=-2$ $\therefore$ slope $m=-2$ Equation of tangent is $y-1=-2\left(x+1\right)$ $\therefore y-1=-2x-2$ $\therefore 2x+y+1=0$ $\therefore$ slope of normal $=\frac{-1}{m}=\frac{1}{2}$ Equation of normal is $y-1=\frac{1}{2}\left(x+1\right)$ $\therefore 2y-2=x+1$ $\therefore x-2y+3=0$	 1 ½ 1 ½ 1
		--	
	b)	Find $\frac{dy}{dx}$ if, $y=x^{\sin x}$	**04**
	Ans	$y=x^{\sin x}$ taking log on both sides, $\therefore \log y=\log x^{\sin x}$ $\therefore \log y=\sin x\log x$ diff.w.r.t.x	 1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	$\therefore \frac{1}{y}\frac{dy}{dx} = \sin x \frac{1}{x} + \log x \cos x$	2
		$\therefore \frac{dy}{dx} = y\left(\frac{\sin x}{x} + \cos x \log x\right)$	½
		$\therefore \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right)$	½
	c)	Find $\frac{dy}{dx}$ if, $y = \tan^{-1}\left(\frac{x}{1+12x^2}\right)$	**04**
	Ans	$y = \tan^{-1}\left(\frac{x}{1+12x^2}\right)$	
		$\therefore y = \tan^{-1}\left(\frac{4x-3x}{1+(4x)(3x)}\right)$	1
		$\therefore y = \tan^{-1}(4x) - \tan^{-1}(3x)$	1
		$\therefore \frac{dy}{dx} = \frac{1}{1+(4x)^2}\frac{d}{dx}(4x) - \frac{1}{1+(3x)^2}\frac{d}{dx}(3x)$	1
		$\therefore \frac{dy}{dx} = \frac{4}{1+16x^2} - \frac{3}{1+9x^2}$	1
	d)	Evaluate, $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}}\,dx$	**04**
	Ans	$\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}}\,dx$	
		Put $\sin^{-1} x = t$	
		$\therefore \frac{1}{\sqrt{1-x^2}}\,dx = dt$	1
		$= \int t^3\,dt$	1
		$= \left(\frac{t^4}{4}\right) + c$	1
		$= \frac{1}{4}(\sin^{-1} x)^4 + c$	1

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4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$	**04**
	Ans	$\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$	
		Put $xe^x = t$	1
		$\therefore e^x(x+1)dx = dt$	
		$= \int \frac{1}{\cos^2 t}dt$	½
		$= \int \sec^2 t\, dt$	1
		$= \tan t + c$	1
		$= \tan(xe^x) + c$	½
		--	
	b)	Evaluate, $\int \frac{dx}{5-4\cos x}$	**04**
	Ans	$\int \frac{dx}{5-4\cos x}$	
		Put $\tan\frac{x}{2} = t$, $dx = \frac{2dt}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$	
		$I = \int \frac{\frac{2dt}{1+t^2}}{5-4\left(\frac{1-t^2}{1+t^2}\right)}$	1
		$= \int \frac{2dt}{5(1+t^2)-4(1-t^2)}$	
		$= 2\int \frac{dt}{5+5t^2-4+4t^2}$	
		$= 2\int \frac{dt}{9t^2+1}$	1
		$= 2\int \frac{dt}{(3t)^2+(1)^2}$ or $= \frac{2}{9}\int \frac{dt}{t^2+\left(\frac{1}{3}\right)^2}$	½

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4.	b)	$=2\frac{1}{1}\tan^{-1}\left(\frac{3t}{1}\right)\frac{1}{3}+c$ or $=\frac{2}{9}\frac{1}{\left(\frac{1}{3}\right)}\tan^{-1}\left(\frac{t}{\frac{1}{3}}\right)+c$	1
		$=\frac{2}{3}\tan^{-1}(3t)+c$	
		$=\frac{2}{3}\tan^{-1}\left(3\tan\frac{x}{2}\right)+c$	½
	c)	Evaluate $\int\tan^{-1}x\,dx$	**04**
	Ans	$\int\tan^{-1}x\,dx$	
		$=\int\tan^{-1}x.1\,dx$	
		$=\tan^{-1}x\int 1dx-\int\left(\left(\int 1dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx+c\right.$	1
		$=x\tan^{-1}x-\int x\frac{1}{1+x^2}dx+c$	1
		$=x\tan^{-1}x-\frac{1}{2}\int\frac{2x}{1+x^2}dx+c$	1
		$=x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+c$	1
	d)	Evaluate, $\int\frac{e^x.\,dx}{\left(e^x-1\right)\left(e^x+1\right)}$	**04**
	Ans	$\int\frac{e^x.\,dx}{\left(e^x-1\right)\left(e^x+1\right)}$	
		Put $e^x=t$	
		$\therefore e^x dx=dt$	1
		$\therefore\int\frac{1}{(t-1)(t+1)}dt$	
		$\frac{1}{(t-1)(t+1)}=\frac{A}{t-1}+\frac{B}{t+1}$	½
		$\therefore 1=A(t+1)+B(t-1)$	
		Put $t=1$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$\therefore A=\frac{1}{2}$	½
		Put $t=-1$	
		$\therefore B=-\frac{1}{2}$	½
		$\therefore \frac{1}{(t-1)(t+1)}=\frac{\frac{1}{2}}{t-1}-\frac{\frac{1}{2}}{t+1}$	
		$\therefore \int\frac{1}{(t-1)(t+1)}dt=\int\left(\frac{\frac{1}{2}}{t-1}-\frac{\frac{1}{2}}{t+1}\right)dt$	
		$=\frac{1}{2}\log(t-1)-\frac{1}{2}\log(t+1)+c$	1
		$=\frac{1}{2}\log(e^x-1)-\frac{1}{2}\log(e^x+1)+c$	½
		OR	
		$\int\frac{e^x.\,dx}{(e^x-1)(e^x+1)}$	
		Put $e^x=t$	1
		$\therefore e^x dx=dt$	
		$\therefore \int\frac{1}{(t-1)(t+1)}dt$	
		$=\int\frac{1}{t^2-1^2}dt$	1
		$=\frac{1}{2(1)}\log\left(\frac{t-1}{t+1}\right)+c$	1
		$=\frac{1}{2}\log\left(\frac{e^x-1}{e^x+1}\right)+c$	1
		--	
	e)	Evaluate, $\int_0^{\pi/2}\frac{dx}{1+\tan x}$	**04**
	Ans	$\int_0^{\pi/2}\frac{dx}{1+\tan x}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$\therefore I=\int_0^{\pi/2}\frac{1}{1+\frac{\sin x}{\cos x}}\,dx$	
		$\therefore I=\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}\,dx$ --------------------(1)	½
		$\therefore I=\int_0^{\pi/2}\frac{\cos\left(\pi/2-x\right)}{\cos\left(\pi/2-x\right)+\sin\left(\pi/2-x\right)}\,dx$	1
		$\therefore I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx$ --------------------(2)	½
		add (1) and (2)	
		$\therefore I+I=\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}\,dx+\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx$	½
		$2I=\int_0^{\pi/2}\frac{\cos x+\sin x}{\cos x+\sin x}\,dx$	
		$2I=\int_0^{\pi/2}1\,dx$	½
		$2I=\left[x\right]_0^{\pi/2}$	½
		$2I=\frac{\pi}{2}-0$	
		$\therefore I=\frac{\pi}{4}$	½
5.		**Attempt any TWO of the following:**	**12**
	a)	Find area bounded by the curve $y^2=4x$ and $x^2=4y$	**06**
	Ans	$y^2=4x$ ---------(1)	
		$x^2=4y$	
		$\therefore y=\frac{x^2}{4}$	
		$\therefore \text{eq}^n.(1)\Rightarrow\left(\frac{x^2}{4}\right)^2=4x$	

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5.	a)	$\frac{x^4}{16}=4x$ $\therefore x^4=64x$ $\therefore x^4-64x=0$ $\therefore x\left(x^3-64\right)=0$ $\therefore x=0,4$	1
		Area $A=\int_a^b\left(y_1-y_2\right)dx$ $\therefore A=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx$	1
		$\therefore A=\int_0^4\left(2x^{\frac{1}{2}}-\frac{x^2}{4}\right)dx$ $\therefore A=\left(\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12}\right)_0^4$	2
		$\therefore A=\left(\frac{2\left(4\right)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{\left(4\right)^3}{12}\right)-0$	1
		$\therefore A=\frac{16}{3}$ or 5.333	1
		--	
	b)	**Attempt the following:**	**06**
	i)	Form a differential equation by eliminating arbitrary constant if $y=A.\cos\left(\log x\right)+B.\sin\left(\log x\right)$	**03**
	Ans	$y=A\cos\left(\log x\right)+B\sin\left(\log x\right)$ $\therefore \frac{dy}{dx}=-A\sin\left(\log x\right)\frac{1}{x}+B\cos\left(\log x\right)\frac{1}{x}$	1
		$\therefore x\frac{dy}{dx}=-A\sin\left(\log x\right)+B\cos\left(\log x\right)$ $\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}(1)=-A\cos\left(\log x\right)\frac{1}{x}-B\sin\left(\log x\right)\frac{1}{x}$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b) i)	$\therefore x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = -A\cos(\log x) - B\sin(\log x)$	
		$\therefore x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = -(A\cos(\log x) + B\sin(\log x))$	½
		$\therefore x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$	
		$\therefore x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$	½
		--	
	b)ii)	Solve, $x(1+y^2)dx + y(1+x^2)dy = 0$	**03**
	Ans	$x(1+y^2)dx + y(1+x^2)dy = 0$	
		$\therefore x(1+y^2)dx = -y(1+x^2)dy$	
		$\therefore \frac{x}{1+x^2}\ dx = \frac{-y}{1+y^2}\ dy$	1
		$\therefore \int \frac{x}{1+x^2}\ dx = -\int \frac{y}{1+y^2}\ dy$	½
		$\therefore \frac{1}{2}\int \frac{2x}{1+x^2}\ dx = -\frac{1}{2}\int \frac{2y}{1+y^2}\ dy$	½
		$\therefore \frac{1}{2}\log(1+x^2) = -\frac{1}{2}\log(1+y^2) + c$	1
		--	
	c)	A particle starting with velocity 6m/s has an acceleration $(1-t^2)$ m/s^2.when does it first come to rest? How far has it then travelled?	**06**
	Ans	Acceleration = $\frac{dv}{dt} = 1-t^2$	
		$\therefore dv = (1-t^2)dt$	½
		$\therefore \int dv = \int (1-t^2)dt$	½
		$\therefore v = t - \frac{t^3}{3} + c$	½
		given $v = 6$ and $t = 0$ $\therefore c = 6$	½
		$\therefore v = t - \frac{t^3}{3} + 6$	

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5.	c)	The particle comes to rest when $v=0$ $\therefore t-\frac{t^3}{3}+6=0$ $\therefore t^3-3t-18=0$ $\therefore t=3\text{sec}$ $\because v=\frac{dx}{dt}$ $\therefore \frac{dx}{dt}=t-\frac{t^3}{3}+6$ $\therefore dx=\left(t-\frac{t^3}{3}+6\right)dt$ $\therefore \int dx=\int\left(t-\frac{t^3}{3}+6\right)\,dt$ $\therefore x=\frac{t^2}{2}-\frac{t^4}{12}+6t+c_1$ $\therefore$ initially $x=0$, $t=0$ $c_1=0$ $\therefore x=\frac{t^2}{2}-\frac{t^4}{12}+6t$ put $t=3$ $\therefore x=\frac{(3)^2}{2}-\frac{(3)^4}{12}+6(3)$ $\therefore x=15.75\ m$	1 ½ ½ ½ ½ 1
6.		**Attempt any <u>TWO</u> of the following:**	**12**
	a) i) Ans	An unbiased coin is tossed 5 times. Find probability of getting three heads. $n=5$, $p=\frac{1}{2}$, $q=\frac{1}{2}$, $r=3$ $\therefore P(r)={}^nC_r\,p^r q^{n-r}$ $\therefore P(3)={}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3}$ $\therefore P(3)=\frac{10}{32}$ or 0.1562	**03** 2 1

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6.	a)ii)	Fit a Poisson's distribution for the following observations	**03**
		x_i: 20, 30, 40, 50, 60, 70 ; f_i: 8, 12, 30, 10, 6, 4	
	Ans	Mean $= m = \dfrac{\sum f_i x_i}{\sum f_i}$	
		$\therefore m = \dfrac{20(8)+30(12)+40(30)+50(10)+60(6)+70(4)}{8+12+30+10+6+4}$	
		$\therefore m = \dfrac{2860}{70} = 40.85$	2
		Poisson distribution is ,	
		$P(x=r) = \dfrac{e^{-m} m^r}{r!}$	
		$\therefore P(r) = \dfrac{e^{-40.85}(40.85)^r}{r!}$	1
	b)	If 2% of the electric bulbs manufactured by a company are defective. Find the probability that in sample of 100 bulbs.	**06**
		(i) 3 are defective	
		(ii) At least two are defective.	
	Ans	$p = 2\% = 0.02$, $n = 100$	
		$\therefore$ mean $m = np$	
		$\therefore m = 100 \times 0.02 = 2$	1
		Poisson's distribution is,	
		$P(r) = \dfrac{e^{-m} \cdot m^r}{r!}$	
		(i) 3 bulbs are defective $\therefore r = 3$	
		$\therefore P(3) = \dfrac{e^{-2}(2)^3}{3!}$	1
		$\therefore P(3) = 0.1804$	1
		(ii) At least two are defective	
		$\therefore P(\text{at least two are defective}) = 1 - \left[P(0) + P(1)\right]$	

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6.	b)	$\therefore P(\text{at least two are defective}) = 1 - \left[\frac{e^{-2}(2)^0}{0!} + \frac{e^{-2}(2)^1}{1!}\right]$	2
		$= 0.5939$	1
	c)	In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution is to be normal, (i) How many students score between 12 and 15 (ii) How many students score above 18 Given Frequency 0 to 0.8 = 0.2881 Frequency 0 to 0.4 = 0.1554 Frequency 0 to 1.6 = 0.4452	**06**
	Ans	Given $\bar{x} = 14$ $\sigma = 2.5$ $N = 1000$	
		$i)\ z = \frac{x - \bar{x}}{\sigma} = \frac{12 - 14}{2.5} = -0.8$	1
		$z = \frac{x - \bar{x}}{\sigma} = \frac{15 - 14}{2.5} = 0.4$	1
		$\therefore p(\text{score between 12 and 15}) = A(-0.8) + A(0.4)$ $= 0.2881 + 0.1554$ $= 0.4435$	1
		$\therefore \text{No.of students} = N \cdot p = 1000 \times 0.4435$ $= 443.5$ *i.e.*, 444	½
		$ii)\ z = \frac{x - \bar{x}}{\sigma} = \frac{18 - 14}{2.5} = 1.6$	1
		$\therefore p(\text{ above 18}) = A(\text{greater than 1.6})$ $= 0.5 - A(1.6)$ $= 0.5 - 0.4452 = 0.0548$	1
		$\therefore \text{No.of students} = N \cdot p$ $= 1000 \times 0.0548 = 54.8$ *i.e.*, 55	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* --- ---	