WINTER – 19 EXAMINATION

Subject Name: STRENGTH OF MATERIALS **Model Answer** **17304**

Important Instructions to examiners:

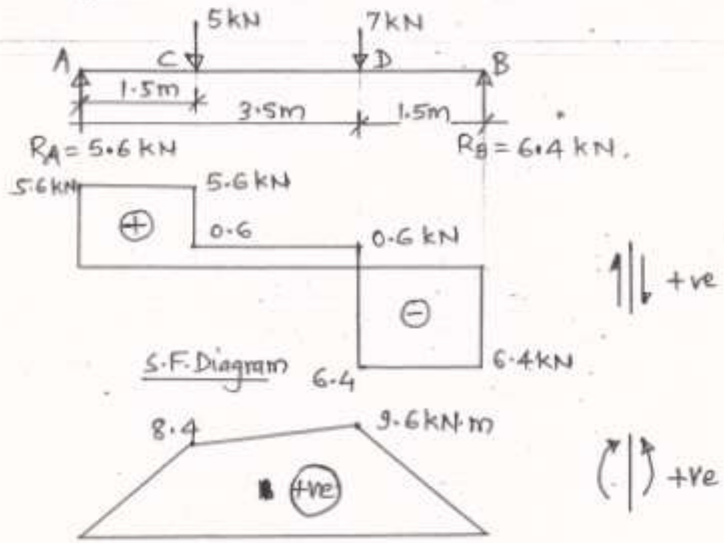
1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.No.	Answer	Marking Scheme
1.	a)	**Attempt any SIX of the following :**	12
	(i)	Define ductility and malleability.	
		Ductility:- It is the property of material by virtue of virtue of which it can be drawn into thin wires on applications of tensile force.	01
		Malleability:- It is the property of material by virtue of which it can be flattened into thin sheets or bent without cracking when hammered.	01
	(ii)	Define principal plane and principal stress.	
		Principal plane:- A plane across which only normal stresses act with no shear stress is called as principal plane.	01
		Principal stress:- The normal stress (direct stress) acting on principal plane is called as principal stress.	01

(iii)	State the parallel axis theorem. Parallel axis theorem:- Statement - The moment of inertia of a lamina about any axis is equal to the M.I. of that lamina about parallel axis passing through its centroid plus the product of area of lamina and the square of perpendicular distance between the two axes. $I_{AB} = I_G + Ah^2$	02
(iv)	Define direct load and eccentric load. Direct load: A load whose line of action acts along the axis of the body is called as direct load. Eccentric load:- A load whose line of action does not coinside with the axis of the body is called as an eccentric load.	01 01
(v)	State torsion equation along with meaning of each term used in it. Torsional Equation, $\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{L}$. Where, T= Torque, I_p= Polar M.I of shaft τ = Max. shear stress, R= Radius of shaft G= Modulus of rigidity of shaft material θ = Angle of twist, L= length of shaft.	01 01
(vi)	Define bulk modulus. Bulk Modulus:- When a body is subjected to like and equal direct stresses along three mutually perpendicular directions, the the ratio of this direct stress to the corresponding volumetric strain is constant within the elastic limit. This ratio is called as bulk modulus.	02

	(vii)	Define hoop stress. State its formula. Hoop stress :- When a cylinder is subjected to internal pressure, the tensile stresses are developed along the circumference of cylinder which are called as hoop stresses or circumferential stresses. formula for hoop stress $f_c = \frac{pd}{2t}$ Where, p = internal pressure. d = diameter of cylinder (Internal) t = thickness of cylindrical shell	01 01
	(viii)	Define middle third rule. Middle third rule :- for a rectangular cross section of column, eccentricity of loading must be within middle third of width or depth, to avoid tensile stresses across the crosssection.	02
1	b)	**Attempt any TWO of the following :**	8
	(i)	A metal rod 24 mm diameter and 2 m long is subjected to an axial pull of 40 kN. If the elongation of the rod is 0.5 mm. Find the stress induced and the value of Young's modulus. Given :- d = 24 mm, L = 2000 mm, $P = 40\times10^3$ N To find :- i) stress (6) and ii) Youngs Modulus (E) Solution :- i) $\text{Stress} = 6 = \frac{P}{A} = \frac{40\times10^3}{(\frac{\pi}{4}\times24^2)} = \frac{40\times10^3}{452.39} = \underline{88.42\ N/mm^2}$ ii) $\text{Young's Modulus} = E = \frac{P\cdot L}{A\cdot\delta L} = \frac{40\times10^3\times2000}{452.39\times0.5}$ $\underline{E = 3.54\times10^5\ N/mm^2}$	02 02

(ii) A simply supported beam of span 5 m carries two point load of 5kN and 7kN at 1.5 m and 3.5 m from the left hand support respectively. Draw shear force and bending moment diagram.



01

01

B.M. Diagram

Reactions: $\Sigma M_A = 0$ +ve.

$5 \times 1.5 + 7 \times 3.5 - R_B \times 5 = 0$

$\therefore R_B = 32/5 = 6.4$ kN

$\Sigma F_y = 0$ ↑ +ve $\quad R_A + R_B - 5 - 7 = 0$.

$\therefore R_A = 12 - 6.4 = 5.6$ kN.

S.F. Values: +ve.

$SF_A = 5.6$ kN

$S.F_C(\text{left}) = 5.6$ kN

$SF_C(\text{right}) = 5.6 - 5 = 0.6$ kN.

$S.F._D(\text{left}) = 0.6$ kN

$S.F_D(\text{right}) = 0.6 - 7 = -6.4$ kN

$S.F_B = -6.4$ kN

B.M. Values: +ve.

$BM_A = 0$

$BM_C = 5.6 \times 1.5 = 8.4$ kN.m 01+01

$BM_D = 5.6 \times 3.5 - 5 \times 2$

$= 9.6$ kN.m.

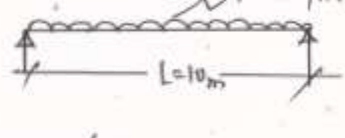
$BM_B = 0$

(iii) A circular beam of 120 mm diameter is simply supported over a span of 10 m and carries a udl of 1000 N/m. Find the maximum bending stress produced.

Given :- S.S. beam, $d = 120$ mm, $L = 10$ m, $w = 1000$ N/m over entire span.

To find :- Max. bending stress.

Soln :-



$$\text{Max. B.M} = M = \frac{wL^2}{8} = \frac{1000 \times 10^2}{8}$$

$$M = 12.5 \times 10^3 \text{ N-m} = 12.5 \times 10^6 \text{ N-mm.}$$ 1+1

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 120^4 = 10.18 \times 10^6 \text{ mm}^4$$

$$y_{max} = \frac{d}{2} = \frac{120}{2} = 60 \text{ mm.}$$ 1

$$\therefore \quad \sigma_{b,max} = \pm \frac{M \cdot y}{I} = \pm \frac{12.5 \times 10^6 \times 60}{10.18 \times 10^6}$$

$$\boxed{\sigma_{b,max} = \pm 73.67 \text{ N/mm}^2}$$ 1

2. Attempt any **FOUR** of the following : 16

a) (i) Define the term modular ratio?

Modular ratio :- The ratio of modulus of elasticities of two different materials is called as modulur ratio. Its value is always more than one. 02

$$\text{Modulur ratio} = m = \frac{E_1}{E_2} \; ; \; E_1 > E_2$$

(ii) State any four assumption made in Euler's theory.

① Column is initially straight and of uniform crosssection.
② Column is long and fails due to buckling only
③ The material of column is perfecty homogeneous & isotopic.
④ The column is subjected to axial load only
⑤ The self weight of column is neglected
⑥ The column is loaded within the elastic limit.

½ Mark for each assumption Max. 2 Marks.

Q. No.	Sub Q.	Answer	Marks
2	b)	A hollow steel tube of 200 mm external diameter and 25 mm thick is 4 m long used as a column. If its one end is fixed and the other end is hinged. Find the load the column can carry. Use Euler's formula and FOS = 2, Take $E = 2 \times 10^5$ N/mm².	
		Given :- Hollow steel tube, $L = 4000$ mm, $D = 200$ mm, $t = 25$ mm One end fixed and other hinged, $E = 2\times10^5$ N/mm²	
		To find :- Column load by Euler's formula.	
		Sol^n :- $d = D - 2t = 200 - 2\times25 = 150$ mm,	
		Effective length $= L_e = \frac{L}{\sqrt{2}} = \frac{4000}{\sqrt{2}} = 2828.43$ mm	01
		For hollow circular section, $I = \frac{\pi}{64}(D^4 - d^4)$	
		$\therefore I = \frac{\pi}{64}(200^4 - 150^4) = 53.69\times10^6$ mm⁴	
		Using Euler's formula, $P_E = \frac{\pi^2 EI}{L_e^2}$	01
		$\therefore P_E = \frac{\pi^2 \times 2\times10^5 \times 53.69\times10^6}{2828.43^2}$	
		$= 13247\times10^3$ N	
		$P_E = 13247$ kN	01
		$\therefore$ Safe load $= P = \frac{P_E}{FoS} = \frac{13247}{2} = 6623.5$ kN	01
2	c)	A rod has a length of 10 m at 10°C and its temperature is raised to 70°C. If the free expansion is prevented. Find the magnitude and nature of stress produced. Take E = 210 kN/mm² and $\alpha = 12 \times 10^{-6}$/°C.	
		Given: $L = 10$ m $= 10000$ mm, $t_1 = 10°c$, $t_2 = 70°c$ $E = 210\times10^3$ N/mm², $\alpha = 12\times10^{-6}$/°c	
		To find :- Temp-stress in magnitude and nature	
		Sol^n :- temp. stress $= \sigma_t = \alpha(t_2 - t_1)E$	01
		$= 12\times10^{-6}\times(70-10)\times210\times10^3$	
		$\sigma_t = 151.2$ N/mm² (Compressive)	02 + 01

2 d) A copper wire 20 mm^2 in cross section and steel wire 30 mm^2 in cross section both 1m long are rigidty connected to plates on either side. They jointly share load of 8 kN. $E_{Steel} = 20 \times 10^5$ N/mm^2. Determine $E_{copper} = 1 \times 10^5$ N/mm^2. Find the stresses produced in each material.

Given :- Copper wire, $A_c = 20 mm^2$, $L_c = 1000 mm$

Steel wire, $A_s = 30 mm^2$, $L_s = 1000$ mm.

Load = $P = 8 kN = 8000$ N.

$E_s = 20 \times 10^5$ N/mm^2

$E_c = 1 \times 10^5$ N/mm^2

To find – stresses in steel and copper wire.

Sol^n :- $m = \frac{E_s}{E_c} = \frac{20}{1} = 20$

$\therefore \sigma_s = m \cdot \sigma_c \rightarrow \quad \sigma_s = 20\,\sigma_c$ — 01

Using the relation –

$P = \sigma_s A_s + \sigma_c \cdot A_c$ — 01

$8 \times 10^3 = 20\sigma_c \times 30 + \sigma_c \times 20 = 620\,\sigma_c$

$\therefore \sigma_c = \frac{8 \times 10^3}{620} = 12.90$ N/mm^2 — 01

$\therefore \sigma_s = 20 \times 12.90 = 258.06$ N/mm^2 — 01

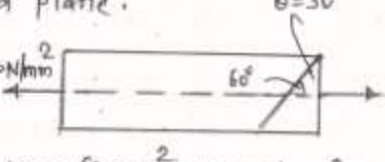
2 e) A bar is subjected to a tensile stress of 100 N/ mm^2. Determine the normal and tangential stresses on a plane making an angle of 60° with the axis of tensile stress.

Given :- $\sigma_x = 100$ N/mm^2, $\theta_1 = 60°$

To find :- σ_n and σ_t on inclined plane.

Sol^n :-

$\theta = 90° - \theta_1 = 90° - 60° = 30°$



i) Normal stress = $\sigma_n = \sigma_x \cdot \cos^2\theta = 100 \times (\cos 30)^2 = 75$ N/mm^2 — 01+01

ii) tangential stress = $\sigma_t = \frac{\sigma_x \cdot \sin 2\theta}{2} = \frac{100 \times \sin(2 \times 30)}{2}$

$\therefore \boxed{\sigma_t = 43.30 \text{ N/mm}^2}$ — 01+01

2 e)

OR

$\sigma_x = 100\ N/mm^2,\ \sigma_y = 0,\ \theta = 30^\circ$

i) Normal stress $= \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$ — 01

$= \frac{100+0}{2} + \frac{100-0}{2}\cos(2\times 30)^\circ$

$\boxed{\sigma_n = 75\ N/mm^2}$ — 01

ii) Tangential stress $= \sigma_t = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$ — 01

$= \frac{100-0}{2}\sin(2\times 30)^\circ$

$= 50\sin 60^\circ$

$\boxed{\sigma_t = 43.30\ N/mm^2}$ — 01

2 f) A gas cylinder of internal diameter 1.2 m and thickness 24 mm is subjected to maximum tensile stress of 90 MPa. Find allowable pressure of gas inside cylinder.

Given: for gas cylinder, $d = 1.2\,m = 1200\,mm,\ t = 24\,mm$

$\sigma_{max} = 90\ N/mm^2$

To find:- internal pressure $= p$

Soln:- $\sigma_{max} = \sigma_c = \frac{pd}{2t}$ — 02

$90 = \frac{p \times 1200}{2 \times 24}$

$\therefore p = \frac{90 \times 2 \times 24}{1200} = 3.6\ N/mm^2$

$\boxed{P = 3.6\ N/mm^2}$ — 02

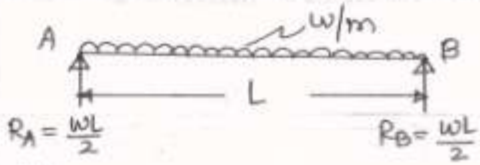
3. **Attempt any FOUR of the following :** — 16

a) Define shear force and bending moment.

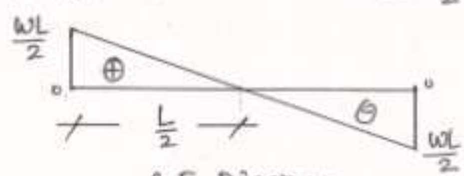
Shear Force :- Shear force at a section of beam is defined as summation of all vertical forces acting on beam on eigther side of the section considered. — 02

Bending Moment :- Bending moment at a section of beam is defined as ~~an~~ algebric summation of moments of all forces about the section considered on either side of section considered. — 02

3 b) Draw SF and BM diagram for a simply supported beam of span L carrying a udl w/unit length over the entire span.

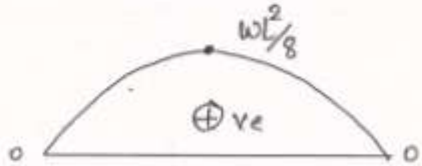


Reactions:-
Due to symmetrical loading
$R_A = R_B = wL/2$



S.F. Diagram

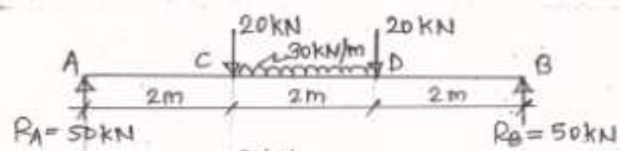
S.F. Values ↑|↓ +ve
$S.F_A = wL/2$
$S.F_B = -wL/2$ 02

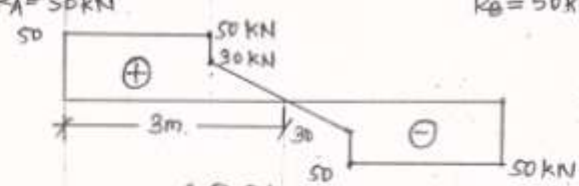


B.M. Diagram

B.M. Values ↻|↺ +ve
$BM_A = BM_c = 0$
$BM_{max} = \frac{wL}{2} \times \frac{L}{2} - \frac{wL}{2} \times \frac{L}{4}$ 02
$= \frac{wL^2}{8}$

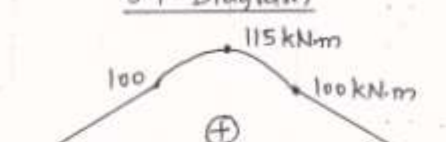
3 c) A simply supported beam of span 6 m carries two point loads of 20 kN each at 2 m and 4 m from left hand support. It also carries udl of 30 kN/m between two point loads. Draw SFD and BMD.





S.F. Diagram

↑|↓ +ve 01



↻|↺ +ve 01

B.M. Diagram.

3 c continued.....

A) Reactions:

$\Sigma M_A = 0$ ↻ +ve.

$20\times2 + 30\times2\times3 + 20\times4 - R_B\times6 = 0$

$\therefore R_B = 300/6 = 50\,kN.$

$\Sigma F_y = 0$ ↑ +ve

$R_A + R_B - 20 - (30\times2) - 20 = 0$

$\therefore R_A = 100 - 50 = 50\,kN.$

B) S.F. Values ↑|↓ +ve.

$SF_A = R_A = 50\,kN.$

$S.F_C(left) = 50\,kN.$

$S.F_C(right) = 50 - 20 = 30\,kN.$ 01

$S.F_D(left) = 30 - (30\times2) = -30\,kN.$

$S.F_D(right) = -30 - 20 = -50\,kN$

$S.F_B = -50\,kN.$

C) B.M. Values ↻|↺ +ve

$BM_A = 0$

$BM_C = R_A\times2 = 50\times2 = 100\,kN.m$

$BM_D = 50\times4 - 20\times2 - 30\times2\times1 = 100\,kN.m$ 01

$BM_B = 0$

$B.M_{max} = 50\times3 - 20\times1 - 30\times1\times0.5 = 115\,kN\text{-}m$
(at midspan)

3 d) A cantilever is loaded as shown in Figure No. 1 Draw SFD and calculate B.M. at point A.

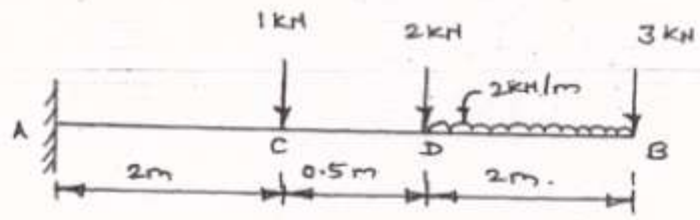
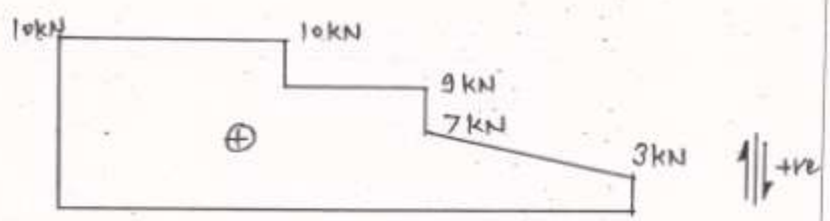


Fig. No. 1



02

S.F. Diagram.

- S.F. Values: +ve.

$S.F_B = 3$ kN.

$S.F_D(\text{right}) = 3+(2\times2) = 7$ kN.

$S.F_D(\text{left}) = 7+2 = 9$ kN.

$S.F_C(\text{right}) = 9$ kN.

$S.F_C(\text{left}) = 9+1 = 10$ kN.

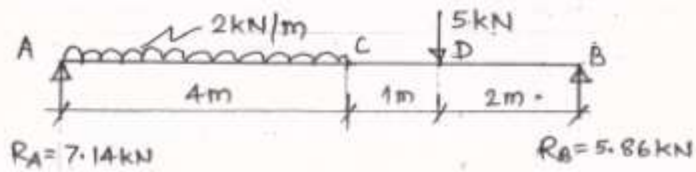
$S.F_A = 10$ kN. 01

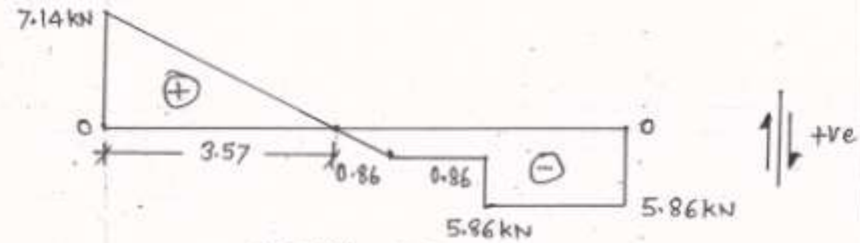
- Bending moment @ A

$$BM_A = -(3\times4.5)-(2\times2\times3.5)-(2\times2.5)+(1\times2)$$ 01

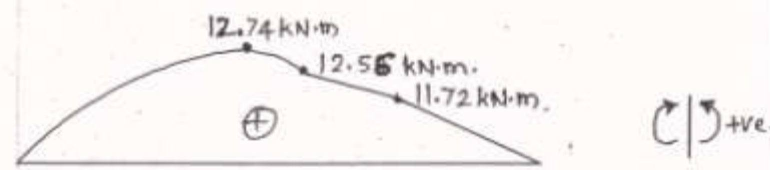
$$\boxed{BM_A = -34.5 \text{ kN.m.}}$$

3	e)	A simply supported beam of span 7 m carries a udl of 2 kN/m over 4 m length from the left support and a point load of 5 kN at 2 m from the right support. Draw S.F. and B.M. diagram.	





S.F. Diagram. 01



B.M. Diagram 01

Ⓐ Reactions, $\Sigma M_A = 0$ +ve.

$2 \times 4 \times 2 + 5 \times 5 - R_B \times 7 = 0$

$41 = 7 R_B \quad \therefore R_B = 41/7 = 5.86$kN

$\Sigma F_y = 0$ ↑ +ve $\therefore R_A + R_B - 2 \times 4 - 5 = 0.$

$\therefore R_A = 13 - 5.86 = 7.14$ kN.

Ⓑ S.F. Values: ↑↓ +ve.

$S.F._A = R_A = 7.14$ kN.

$S.F_C = 7.14 - (2 \times 4) = 0.86$ kN.

$S.F_D(\text{left}) = -0.86$ kN.

$S.F_D(\text{right}) = -0.86 - 5 = -5.86$ kN 01

$S.F._B = -5.86$ kN.

Position of point of contrashear.

from S.F. diagram, $\frac{4-d}{d} = \frac{7.14}{0.86}$

$d = 0.43\,m$ from point c

and at a dist. 3.57 m. from point 'A'

© B.M. Calculations: C|↻ +ve

$BM_A = 0$

$BM_C = 7.14 \times 4 - 2 \times 4 \times 2 = 12.56$ kN.m. 01

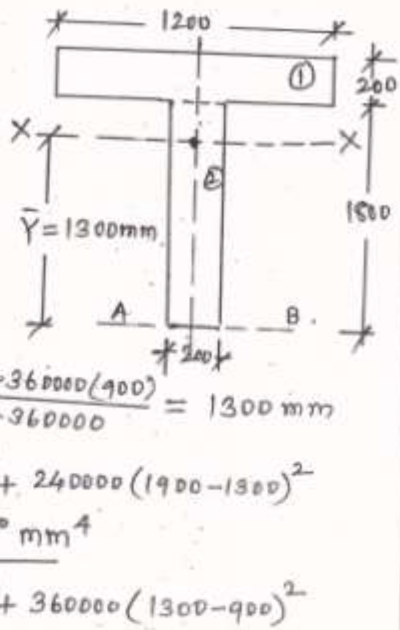
$B.M_D = 7.14 \times 5 - 2 \times 4 \times 3 = 11.7$ kN.m.

OR $BM_D = R_B \times 2 = 5.86 \times 2 = 11.72$ kN.m.

$B.M\,max = 7.14 \times 3.57 - 2 \times \frac{3.57^2}{2} = 12.74$ kN.m.

3 f) Find the M.I. of a 'T' section about the centroidal axis xx. Top flange is 1200×200 mm and web is $1800\ mm \times 200\ mm$. Total height is 2000 mm.

from fig:-



$a_1 = 1200 \times 200 = 240000\ mm^2$

$y_1 = 1800 + \frac{200}{2} = 1900\,mm.$

$a_2 = 200 \times 1800 = 360000\ mm^2$

$y_2 = 1800/2 = 900\ mm.$

Position of Centroid.

$\bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{240000(1900)+360000(900)}{240000+360000} = 1300\ mm$ 01

$(I_{xx})_1 = I_G + Ah^2 = \frac{1200 \times 200^3}{12} + 240000\,(1900-1300)^2$ 01

$(I_{xx})_1 = 8.72 \times 10^{10}\ mm^4$

$(I_{xx})_2 = I_G + Ah^2 = \frac{200 \times 1800^3}{12} + 360000\,(1300-900)^2$ 01

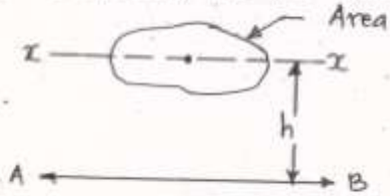
$(I_{xx})_2 = 15.48 \times 10^{10}\ mm^4$

$\therefore\ I_{xx} = (I_{xx})_1 + (I_{xx})_2 = 8.72 \times 10^{10} + 15.48 \times 10^{10} = 24.2 \times 10^{10}\ mm^4$ 01

4.		**Attempt any <u>FOUR</u> of the following :**	16
	a)	State with neat sketches parallel axis theorem and perpendicular axis theorem.	

i) Parallel axis theorem :-

Statement :- The moment of inertia of a lamina about any axis is equal to the M.I. of that lamina about parallel axis passing through its centroid plus the product of area of lamina and square of the distance between the two axes. 01

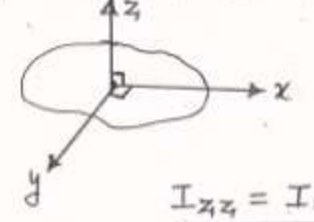


01

$$I_{AB} = I_{xx} + Ah^2$$

ii) Perpendicular axis theorem :-

Statement :- Moment of inertia of a lamina about an axis perpendicular to the lamina and passing through the centroid is equal to the sum of moments of inertia of the lamina about two mutually perpendicular axes passing through the centroid of the lamina. 01



$$I_{zz} = I_{xx} + I_{yy}$$

01

4 b) A hollow C.I. pipe, with external diameter 100 mm and thickness of metal 10 mm is used as a strut. Calculate the moment of inertia and radius of gyration about its diameter.

Given : for hollow C.I. pipe, $D = 100$ mm, $t = 10$ mm.

To find :- I_{xx} and radius of gyration (K)

Soln :- $d = D - 2t = 100 - 2\times10 = 80$ mm.

• for hollow circular section $I_{xx} = \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}(100^4 - 80^4)$

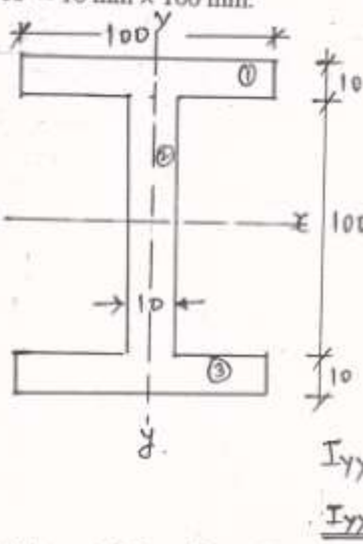
$I_{xx} = 2.898\times10^6\ mm^4$ — 01

• c/s Area $= A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(100^2 - 80^2) = 2827.43\ mm^2$ — 01

• Radius of gyration $= K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{2.898\times10^6}{2827.43}}$ — 01+01

$\boxed{K_{xx} = 32.01\ mm}$

4 c) A symmetrical I-section has the following dimensions. Calculate polar M.I. of the section flanges = 100 mm × 10 mm, web = 10 mm × 100 mm.



• M.I. @ xx-axis :-

$I_{xx} = \frac{BD^3 - bd^3}{12}$

$= \frac{(100\times120^3 - 90\times100^3)}{12}$

$I_{xx} = 6.9\times10^6\ mm^4$ — 01

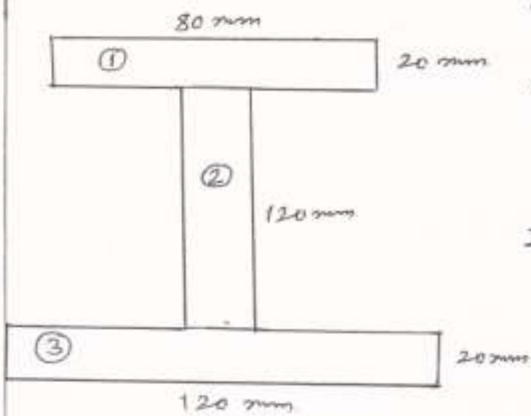
• M.I. @ YY-axis :-

$I_{yy} = \frac{10\times100^3}{12} + \frac{100\times10^3}{12} + \frac{10\times100^3}{12}$ — 01

$I_{yy} = 1.675\times10^6\ mm^4$

• Polar M.I. $= I_p = I_{xx} + I_{yy} = 6.9\times10^6 + 1.675\times10^6 = 8.575\times10^6\ mm^4$ — 01+01

4 d. An I-section consist of top flange 80 × 20 mm, web 120 mm × 20 mm and bottom flange 120 × 20 mm. Calculate moment of inertia about xx-axis.



$A_1 = 1600 \quad A_2 = 2400 \quad A_3 = 2400$

$Y_1 = 150 \quad Y_2 = 80 \quad Y_3 = 10$

$$\bar{y} = \frac{(1600 \times 150) + (2400 \times 80) + (2400 \times 10)}{(1600 + 2400 + 2400)}$$

$= 71.25$ mm. 01

$$I_{x_1} = \frac{1}{12}(80)(20)^3 + (1600)(88.75 - 10)^2$$

$= 9.98 \times 10^6$

$$I_{x_2} = \frac{1}{12}(20)(120)^3 + (2400)(80 - 71.25)^2 \quad 02$$

$= 3.06 \times 10^6$

$$I_{x_3} = \frac{1}{12}(120)(20)^3 + (2400)(71.25 - 10)^2$$

$= 9.08 \times 10^6$

$I_{xx} = I_{x_1} + I_{x_2} + I_{x_3} = 9.98 \times 10^6 + 3.06 \times 10^6 + 9.08 \times 10^6 = 22.12 \times 10^6 \text{ mm}^4$ 01

4 e. State any four assumption made in the theory of simple bending.

<u>Assumptions made in theory of simple bending:-</u>

1) The beam is straight before loading and is of uniform cross-section throughout.
2) The beam material is stressed within elastic limit and thus obaye Hook's law.
3) The transverse sections remain plane before and after bending.
4) Each layer of the beam is free to expand or contract independently of the layer above or below it
5) The material of beam is homogeneous and isotropic
6) The beam is subjected to pure bending only.

1 Mark for each assumption Max. 4 Marks.

4 f. A beam 100 mm wide and 250 mm deep is subjected to a shear force of 40 kN at a certain section. Find the maximum shear stress and draw the shear stress variation diagram.

Given:- beam, $b = 100\,mm$, $d = 250\,mm$.

$S.F = 40\,kN = 40 \times 10^3\,N$.

To find:- i) q_{max} ii) Shear stress variation diagram.

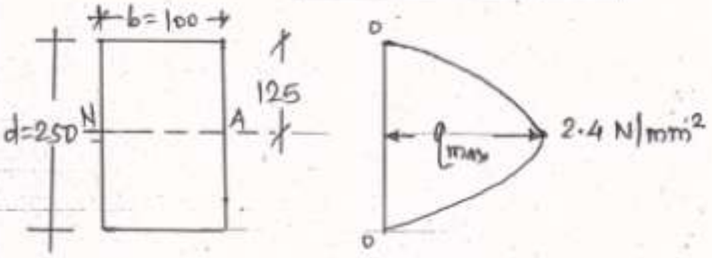
Soln:-

for rectangular section $q_{avg} = \frac{S.F}{C/s\ Area} = \frac{S.F}{b \times d}$

$\therefore q_{avg} = \frac{40 \times 10^3}{100 \times 250} = 1.6\,N/mm^2$ 01

& $q_{max} = 1.5 \times q_{avg} = 1.5 \times 1.6$ 01+01

$\therefore \boxed{q_{max} = 2.4\,N/mm^2}$



C/s of beam Shear stress variation diagram. 01

5. Attempt any FOUR of the following : 16

a) A timber beam has a cross - section 120 mm × 200 mm. It is simply supported over a span of 4 m and carries a udl of 1 kN/m over the entire span. Calculate the maximum bending stress induced in the beam and the radius of curvature to which the beam will bend at that section.

Given, timber beam, $b = 120\,mm$, $d = 200\,mm$

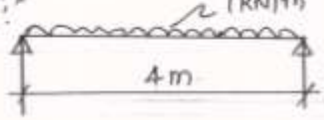
S.S. span = 4 m, UDL = 1 kN/m over entire span.

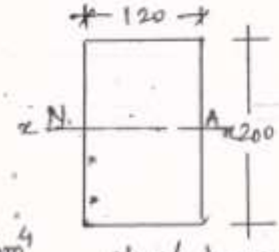
To find:- i) Max. bending stress ($\sigma_{b,max}$)

ii) Radius of curvature (R).

(Note:- As value of E is not given, student can assume any value of E, hence appropriate marks should be given)

5 a) Solⁿ:-





c/s of beam

$M.I = I = \frac{bd^3}{12} = \frac{120 \times 200^3}{12} = 80 \times 10^6 \, mm^4$

$y_{max} = \frac{d}{2} = \frac{200}{2} = 100 \, mm.$

$B.M_{max} = \frac{wL^2}{8} = \frac{1 \times 4^2}{8} = 2 \, kN\cdot m = 2 \times 10^6 \, N\cdot mm.$ 01

Flexural Equation $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ 01

* Max. bending stress $= \sigma_{b,max} = \frac{M . y_{max}}{I}$

$= \frac{2 \times 10^6 \times 100}{80 \times 10^6}$

$\therefore \sigma_{b,max} = 2.50 \, N/mm^2$ 01

* Radius of curvature, $R = \frac{EI}{M}$ OR $\frac{E \cdot y}{R}$

Assumming $E = 0.12 \times 10^5 \, N/mm^2$ for wood.

$R = \frac{0.12 \times 10^5 \times 80 \times 10^6}{2 \times 10^6}$

$R = 480 \times 10^3 \, mm$ 01

5 b) Sketch the shear stress distribution diagram for a rectangular beam of 600 mm × 200 mm deep subjected to shear force of 20 kN.

Given:- Rectangular beam — $b = 600 \, mm$, $d = 200 \, mm$.

$S.F = 20 \times 10^3 \, N$.

To find:- shear stress distribution

Solⁿ:- for rectangular beam, $q_{avg} = \frac{S.F}{b \times d} = \frac{20 \times 10^3}{600 \times 200} = 0.167 \, N/mm^2$ 01

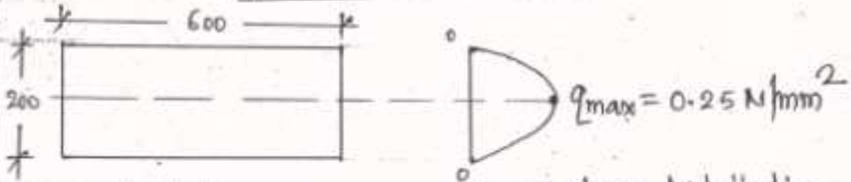
5 b continued.....

For rectangular beam section –

$$q_{max} = 1.5\, q_{average} = 1.5 \times \frac{S.F}{c/s\ Area} = 1.5 \times \frac{S.F}{b \times d}$$

$$= 1.5 \times \frac{20 \times 10^3}{(600 \times 200)} = 1.5 \times 0.167$$ 01+01

$$\therefore \underline{q_{max} = 0.25\ N/mm^2}$$



c/s of beam — Shear stress distribution 01

5 c) A hollow circular column having external and internal diameter of 40 cm and 30 cm respectively carries a vertical load of 150 kN at the outer edge of the column. Calculate the maximum and minimum intensities of stress in the section.

Given :- For hollow circular column, $D = 400mm$, $d = 300mm$.

$P = 150 \times 10^3 N$, $e = D/2 = 200mm$.

To find :- σ_{max} and σ_{min}.

Soln :- for hollow circular section :–

$$A = \frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4}(400^2 - 300^2) = 54977.87\ mm^2$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(400^4 - 300^4) = 8.59 \times 10^8\ mm^4$$

$$y = D/2 = 400/2 = 200\ mm.$$

- Direct stress $= \sigma_0 = \frac{P}{A} = \frac{150 \times 10^3}{54977.87} = \underline{2.73\ N/mm^2}$ 01
- Bending stress $= \sigma_b = \pm \frac{P \cdot e \cdot y}{I} = \frac{150 \times 10^3 \times 200 \times 200}{8.59 \times 10^8}$

$$\sigma_b = \pm 6.98\ N/mm^2$$ 02

- $\sigma_{max} = \sigma_0 + \sigma_b = 2.73 + 6.98 = \underline{9.71\ N/mm^2\ (C)}$
- $\sigma_{min} = \sigma_0 - \sigma_b = 2.73 - 6.98 = -4.25 = \underline{4.25\ N/mm^2\ (T)}$ 01

5 d) A M.S. link as shown in Figure No. 2 transmit a pull of 80 kN. Find the dimensions b and t, if $b = 3t$. Assume the permissible tensile stress as 70 MPa.

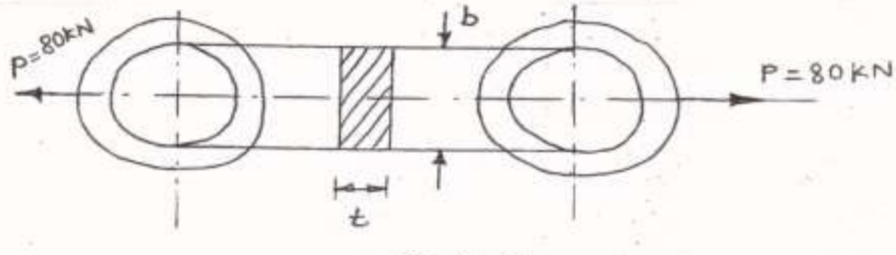


Fig. No. 2

$$\text{Stress} = \frac{\text{Force}}{\text{c/s Area}} = \frac{80\times10^3}{t\times b} = \frac{80\times10^3}{t\times 3t}$$ 01

$$70 = \frac{80\times10^3}{3t^2}$$

$$t^2 = \frac{80\times10^3}{70\times3} = 380.95$$ 01

$t = 19.52$ mm say 20 mm. 01

$b = 3t = 3(19.52) = 58.56$ mm say 60 mm 01

5 e) A clamp made up of rectangular cross-section 30 × 10 mm as shown in Figure No. 3 is subjected to a force of 2.5 kN. Find the stresses induced at section A-A.

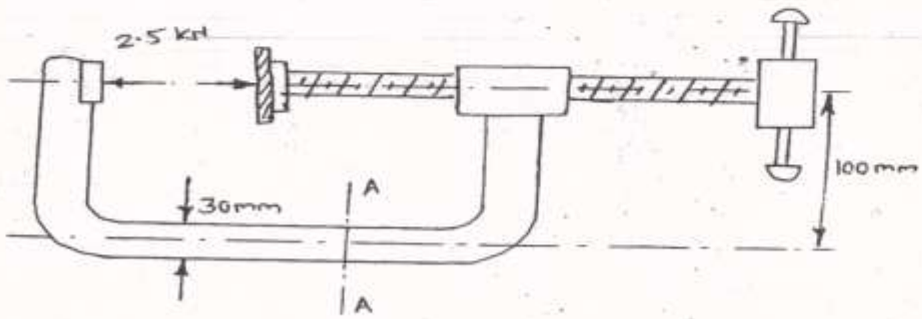


Fig. No. 3

Given :- Rectangular c/s $b = 10\,mm$, $d = 30\,mm$.

$P = 2.5 \times 10^3\,N$, $e = 100\,mm$.

To find :- σ_{max} and σ_{min} at section AA

Soln :-

- Direct stress $= \sigma_0 = \frac{P}{b \times d} = \frac{2.5 \times 10^3}{10 \times 30} = 8.33\,N/mm^2$ — 01 —

$M.I = I = \frac{bd^3}{12} = \frac{10 \times 30^3}{12} = 22500\,mm^4$

$y_{max} = \frac{d}{2} = \frac{30}{2} = 15\,mm.$ 01

- Bending stress $= \sigma_b = \pm \frac{P \cdot e \cdot y_{max}}{I}$

$\therefore \sigma_b = \pm \frac{2.5 \times 10^3 \times 100 \times 15}{22500}$

$\sigma_b = \pm 166.67\,N/mm^2$ 01

(Tensile)

$\sigma_{max} = \sigma_0 + \sigma_b = 8.33 + 166.67 = 175\,N/mm^2\,(T)$

$\sigma_{min} = \sigma_0 - \sigma_b = 8.33 - 166.67 = -158.34\,N/mm^2$ 01

$\sigma_{min} = 158.34\,N/mm^2\,(C)$

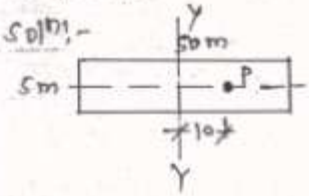
(Comp)

5 f) A mild steel flat 50 mm wide and 5 mm thick is subjected to load 'P' acting in the plane bisecting the thickness at a point 10 mm away from the centroid of the section. If the tensile stress is not to exceed 150 MPa, calculate the magnitude of 'P'.

Given: $b = 50\,mm$, $t = 5\,mm$, $e = 10\,mm$, $\sigma_t = 150\,N/mm^2$

To find: Tensile load P

Soln:-



Eccentricity @ YY- axis -

$A = 50 \times 5 = 250\ mm^2$

$I_{yy} = \frac{5 \times 50^3}{12} = 5.21 \times 10^4\ mm^4$

$y_{max} = 50/2 = 25\ mm.$ — 01

- Direct stress = $\sigma_0 = \frac{P}{A} = \frac{P}{250} = \underline{4 \times 10^{-3} P}\ N/mm^2$ — 01

- Bending stress = $\sigma_b = \pm \frac{P \times e \times y_{max}}{I}$

$= \pm \frac{P \times 10 \times 25}{5.21 \times 10^4}$

$\underline{\sigma_b = \pm 4.8 \times 10^{-3} P}\ N/mm^2$ — 01

- $\sigma_{max} = \sigma_0 + \sigma_b$

$150 = 4 \times 10^{-3} P + 4.8 \times 10^{-3} P = 8.8 \times 10^{-3} P$

$\therefore P = 150 / (8.8 \times 10^{-3}) = 17045.45\,N$

$\therefore \underline{P = 17.045\ kN \cong 17.05\ kN}$ — 01

6. **Attempt any FOUR of the following :** **16**

a) State assumption in theory of pure torsion.

Assumption in theory of pure torsion :- 1 Marks for each assumption Max. 4

1) The material of shaft is homogeneous and isotropic.
2) The section which is plane before twist remains plane after twist.
3) The angle of twist is uniform thoughout the length of shaft.
4) The section of shaft is uniform throughout the length.
5) The shaft is loaded within the elastic limit

6 b) Find the torque that can be applied to a shaft of 100 mm in diameter, if the permissible angle of twist is 2.75° in a length of 6 m. Take C = 80 kN/mm².

Given, for solid shaft, $d = 100$ mm, $L = 6000$ mm.
$\theta = 2.75^\circ$, $C = 80 \times 10^3$ N/mm²

To find :- Torque

Soln:- $\theta = 2.75^\circ = 2.75 \times \frac{\pi}{180} = 0.048$ radians. 01

for shaft, $I_p = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 100^4 = 9.82 \times 10^6 \, mm^4$ 01

Using the relation $\frac{T}{I_p} = \frac{G\theta}{L}$

$\therefore T = \frac{G\theta \times I_p}{L} = \frac{80 \times 10^3 \times 0.048 \times 9.82 \times 10^6}{6000}$ 01

$\therefore T = 6.28 \times 10^6$ N·mm

$T = 6.28$ kN·m 01

6 c) Calculate the suitable diameter of a solid shaft to transmit 220 KW at 150 rpm if the permissible shear stress is 68 MPa.

Given:- Solid shaft, $P = 220\,kW$, $N = 150$, $\sigma_s = 68\,N/mm^2$

To find- diameter - (d).

Soln:- Power, $P = \frac{2\pi N T_{avg}}{60000}$

$$\therefore T_{avg} = \frac{P \times 60000}{2\pi \times 150} = \frac{220 \times 60000}{2\pi \times 150}$$

$$= 14 \times 10^3\,N\cdot m = 14 \times 10^6\,N\cdot mm.$$ 01

Considering $T_{avg} = T_{max} = 14 \times 10^6\,N\cdot mm$.

$$\frac{T}{I_p} = \frac{\sigma_s}{R}$$

$$I_p = \frac{\pi}{32} D^4 = 0.098 D^4, \quad R = 0.5D.$$ 01

$$\therefore \frac{14 \times 10^6}{0.098 D^4} = \frac{68}{0.5D}$$ 01

$$D^3 = \frac{0.5 \times 14 \times 10^6}{0.098 \times 68} = 1.05 \times 10^6$$

$$\therefore D = 101.65\,mm \text{ Say } 110\,mm$$ 01

6 d) A shaft is required to transmit 20 KW at 150 rpm. The maximum torque may exceed average torque by 40%. Determine the diameter. of shaft, if the shear stress is not to exceed 50 MPa.

Given:- For solid shaft, $P = 20\,kW$, $N = 150$,
$T_{max} = 1.4\,T_{avg}$, $\sigma_s = 50\,N/mm^2$

To find:- diameter (d)

Soln:- Power $= P = \frac{2\pi N T_{avg}}{60000}$

$$T_{avg} = \frac{20 \times 60000}{2\pi \times 150} = 1273.24\,N\text{-}m = 1273.24 \times 10^3\,N\text{-}mm$$ 01

		$T_{max} = 1.4\,T_{avg} = 1.4 \times 1273.24 \times 10^3 = 1.78 \times 10^6$ N.mm	
		for solid shaft, $I_p = \frac{\pi}{32} D^4 = 0.098 D^4$, $R = 0.5D$	01
		$\frac{T}{I_p} = \frac{\sigma_s}{R}$	
		$\therefore \frac{1.78 \times 10^6}{0.098 D^4} = \frac{50}{0.5D}$	01
		$D^3 = \frac{1.78 \times 10^6 \times 0.5}{0.098 \times 50} = 1.82 \times 10^5$	
		$\therefore$ D = 56.63 mm Say 60 mm	01
6	e)	Find the torsional moment of resistance for a hollow circular shaft of 225 mm external diameter and 220 mm internal diameter. If the permissible shear stress is 60 MPa.	
		Given, for hollow circular shaft, D = 225 mm, d = 220 mm, $\sigma_s = 60$ N/mm^2	
		To find:- Torsional moment of resistance.	
		Solution-	
		for hollow circular section,	
		$I_p = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(225^4 - 220^4) = 21.63 \times 10^6$ mm^4	01
		$R = D/2 = \frac{225}{2} = 112.5$ mm	01
		Using the relation —	
		$\frac{T}{I_p} = \frac{\sigma_s}{R}$	01
		$\therefore T = \frac{\sigma_s}{R} \times I_p = \frac{60}{112.5} \times 21.63 \times 10^6$	
		$\therefore T = 11.54 \times 10^6$ N.mm	
		T = 11.54 kNm.	01

Q. No.	Sub Q.	Answer	Marks
6	f)	(i) Define - section modulus Section Modulus:- The ratio of moment of inertia about neutral axis to the distance between neutral axis to the outermost layer of section is called as section modulus & is denoted by "Z" Mathematically, $Z = \frac{I}{y_{max}}$.	02
		(ii) Define torque and state its S.I. units. Torque:- When a force is applied on perifери or circumference and in the plane of crosssection, it creates a twisting moment & this twisting moment is called as torque. Torque is equal to the product of force and the perpendicular distance the applied force and axis of the member.	01
		Unit of torque is Nmm or kN-m.	01