WINTER– 2019 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 17301**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any TEN of the following:**	**10**
	a)	Find the point on the curve $y=7x-3x^2$ where the inclination of the tangent is 45^0	**02**
	Ans	$y=7x-3x^2$	
		$\therefore \frac{dy}{dx}=7-6x$	½
		$\therefore m=7-6x$	
		$\therefore \tan 45^0=7-6x$	
		$\therefore 1=7-6x$	½
		$\therefore x=1$	½
		$\therefore y=7(1)-3(1)^2=4$	½
		$\therefore (x,y)=(1,4)$	
		--	
	b)	Evaluate $\int \frac{\cos(\log x)}{x}dx$	**02**
	Ans	$\int \frac{\cos(\log x)}{x}dx$	
		Put $\log x=t$	
		$\therefore \frac{1}{x}dx=dt$	½
		$\therefore \int \cos t\ dt$	
		$=\sin t+c$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b)	$= \sin(\log x) + c$	½
	c)	Find the radius of curvature of the curve $y = x^3$ at $(1,1)$	**02**
	Ans	$y = x^3$	
		$\frac{dy}{dx} = 3x^2$	
		$\frac{d^2y}{dx^2} = 6x$	½
		at $(1,1)$	
		$\frac{dy}{dx} = 3(1)^2 = 3$	
		$\frac{d^2y}{dx^2} = 6(1) = 6$	½
		$\therefore$ Radius of curvature is $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho = \frac{\left[1+(3)^2\right]^{\frac{3}{2}}}{6}$	½
		$\therefore \rho = 5.27$	½
	d)	Evaluate $\int_1^2 \frac{dx}{2x+5}$	**02**
	Ans	$\int_1^2 \frac{dx}{2x+5}$	
		$= \frac{1}{2}\left[\log(2x+5)\right]_1^2$	1
		$= \frac{1}{2}\left[\log(2(2)+5) - \log(2(1)+5)\right]$	½
		$= \frac{1}{2}\left[\log 9 - \log 7\right]$ or $\frac{1}{2}\log\left(\frac{9}{7}\right)$	½

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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	e)	Find order and degree of the differential equation $\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{\frac{3}{2}}$	**02**
	Ans	$\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{\frac{3}{2}}$ Squaring on both sides $\left(\frac{d^2y}{dx^2}\right)^2=\left(y+\frac{dy}{dx}\right)^3$ $\therefore Order=2$ $\therefore Degree=2$	 1 1
	f)	Evaluate $\int\left(e^x+x^e+e^e\right)dx$	**02**
	Ans	$\int\left(e^x+x^e+e^e\right)dx$ $=e^x+\frac{x^{e+1}}{e+1}+e^ex+c$	 2
	g)	Verify that $y=\log x$ is a solution of $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	**02**
	Ans	$y=\log x$ $\therefore \frac{dy}{dx}=\frac{1}{x}$ $\therefore \frac{d^2y}{dx^2}=-\frac{1}{x^2}$ $\therefore x\frac{d^2y}{dx^2}+y$ $=x\left(-\frac{1}{x^2}\right)+\frac{1}{x}$ $=-\frac{1}{x}+\frac{1}{x}$ $=0$ **OR** $y=\log x$	 ½ ½ ½ ½

Q. No.	Sub Q. N.	Answers	Marking Scheme
1.	g)	$\therefore \frac{dy}{dx}=\frac{1}{x}$	½
		$\therefore x\frac{dy}{dx}=1$	½
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}(1)=0$	½
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	½
	h)	Evaluate $\int \log x\, dx$	**02**
	Ans	$\int \log x\, dx$	
		$=\int \log x.1\, dx$	½
		$=\log x\int 1dx-\int\left(\int 1dx.\frac{d(\log x)}{dx}\right)$	½
		$=x.\log x-\int x.\frac{1}{x}\, dx$	½
		$=x.\log x-\int 1dx$	
		$=x.\log x-x+c$	½
	i)	Verify that $y=\cos x$ is a solution of $\frac{d^2y}{dx^2}+y=0$	**02**
	Ans	$y=\cos x$	
		$\therefore \frac{dy}{dx}=-\sin x$	½
		$\therefore \frac{d^2y}{dx^2}=-\cos x$	½
		$\therefore \frac{d^2y}{dx^2}+y$	
		$=-\cos x+\cos x$	½
		$=0$	½
		OR	
		$y=\cos x$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	i)	$\therefore \frac{dy}{dx} = -\sin x$	½
		$\therefore \frac{d^2y}{dx^2} = -\cos x$	½
		$\therefore \frac{d^2y}{dx^2} = -y$	½
		$\therefore \frac{d^2y}{dx^2} + y = 0$	½
	j)	In a sample of 100 bulbs, if 5% of electric bulbs manufactured by a company are defective. Using Poission distribution find the mean.	**02**
	Ans	$n = 100$, $p = 5\% = 0.05$	
		$\therefore$ mean $m = np$	
		$\therefore m = 100 \times 0.05$	1
		$\therefore m = 5$	1
	k)	An unbaised coin is tossed 6 times. Find the probability of getting 2 heads.	**02**
	Ans	Given $n = 6$, $r = 2$	
		$p = \frac{1}{2}$, $q = \frac{1}{2}$	
		$\because p(r) = {}^nC_r (p)^r (q)^{n-r}$	
		$\therefore p(2) = {}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2}$	1
		$\therefore p(2) = \frac{15}{64}$ or 0.2343	1
	l)	Evaluate: $\int \frac{1}{x+\sqrt{x}}\, dx$	**02**
	Ans	$\int \frac{1}{x+\sqrt{x}}\, dx$	
		$= \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}\, dx$	½
		Put $\sqrt{x} + 1 = t$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	l)	$\therefore \frac{1}{2\sqrt{x}}\ dx = dt$	
		$\therefore \frac{1}{\sqrt{x}}\ dx = 2dt$	½
		$\therefore \int \frac{1}{t}\ 2dt$	
		$= 2\int \frac{1}{t}\ dt$	
		$= 2\log t + c$	½
		$= 2\log\left(\sqrt{x}+1\right)+c$	½
		--	
	m)	Evaluate: $\int \frac{1}{x\log x}\ dx$	**02**
	Ans	$\int \frac{1}{x\log x}\ dx$	
		Put $\log x = t$	
		$\therefore \frac{1}{x}dx = dt$	½
		$= \int \frac{1}{t}\ dt$	½
		$= \log t + c$	½
		$= \log\left(\log x\right)+c$	½
		--	
	n)	Evaluate: $\int_2^4 \frac{dx}{2x+3}$	**02**
	Ans	$\int_2^4 \frac{dx}{2x+3}$	
		$= \frac{1}{2}\left[\log\left(2x+3\right)\right]_2^4$	1
		$= \frac{1}{2}\left[\log\left(2\left(4\right)+3\right)-\log(2\left(2\right)+3)\right]$	½
		$= \frac{1}{2}\left[\log 11 - \log 7\right]$ or $= \frac{1}{2}\log\left(\frac{11}{7}\right)$	½
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Q. No.	Sub Q.N.	Answers	Marking Scheme
2.		**Attempt any FOUR of the following:**	**16**
	a)	Evaluate: $\int \frac{e^x(x+1)}{\cos^2(xe^x)}\,dx$	
	Ans	$\int \frac{e^x(x+1)}{\cos^2(xe^x)}\,dx$	
		Put $xe^x = t$	
		$\therefore xe^x + e^x(1)\,dx = dt$	
		$\therefore e^x(x+1)\,dx = dt$	1
		$=\int \frac{1}{\cos^2 t}\,dt$	1
		$=\int \sec^2 t\,dt$	½
		$= \tan t + c$	1
		$= \tan(xe^x) + c$	½
	b)	Evaluate $\int x\tan^{-1} x dx$	**04**
	Ans	$\int x\tan^{-1} x dx$	
		$= \tan^{-1} x\int x\,dx - \int\left(\int x\,dx\right)\frac{d}{dx}\left(\tan^{-1} x\right)dx$	1
		$= \tan^{-1} x\cdot\frac{x^2}{2} - \int\frac{x^2}{2}\cdot\frac{1}{1+x^2}dx$	1
		$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\int\frac{x^2}{1+x^2}dx$	
		$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$	1
		$= \frac{x^2\tan^{-1} x}{2} - \frac{1}{2}\left(x-\tan^{-1} x\right)+c$	1
	c)	Evaluate: $\int\frac{x^2+6x-8}{x^3-4x}\,dx$	**04**
	Ans	$\int\frac{x^2+6x-8}{x^3-4x}\,dx = \int\frac{x^2+6x-8}{x(x-2)(x+2)}\,dx$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	$\therefore \frac{x^2+6x-8}{x(x-2)(x+2)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+2}$	½
		$x^2+6x-8=(x-2)(x+2)A+x(x+2)B+x(x-2)C$	
		Put $x=0$	
		$-8=-4A$	
		$\therefore A=2$	½
		Put $x=2$	
		$8=8B$	½
		$\therefore B=1$	
		Put $x=-2$	
		$-16=8c$	½
		$\therefore c=-2$	
		$\therefore \frac{x^2+6x-8}{x(x-2)(x+2)}=\frac{2}{x}+\frac{1}{x-2}-\frac{2}{x+2}$	
		$\therefore \int \frac{x^2+6x-8}{x(x-2)(x+2)}dx=\int\left(\frac{2}{x}+\frac{1}{x-2}-\frac{2}{x+2}\right)dx$	
		$=2\log x+\log(x-2)-2\log(x+2)+c$	2
		--	
	d)	Find maxima and minima of x^3-18x^2+96x	**04**
	Ans	Let $y=x^3-18x^2+96x$	
		$\therefore \frac{dy}{dx}=3x^2-36x+96$	½
		$\therefore \frac{d^2y}{dx^2}=6x-36$	½
		Consider $\frac{dy}{dx}=0$	
		$3x^2-36x+96=0$	1
		$\therefore x=4$ or $x=8$	
		at $x=4$	
		$\frac{d^2y}{dx^2}=6(4)-36=-12<0$	½
		$\therefore y$ is maximum at $x=4$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$y_{\max}=(4)^3-18(4)^2+96=160$	½
		$at\ \ x=8$	
		$\dfrac{d^2y}{dx^2}=6(8)-36=12>0$	½
		$\therefore y$ is minimum at $x=8$	
		$y_{\min}=(8)^3-18(8)^2+96=128$	½
		--	
	e)	Find the equation of the tangent and normal to the curve $2x^2-xy+3y^2=18$ at (3,1)	**04**
	Ans	$2x^2-xy+3y^2=18$	
		$\therefore 4x-\left(x\dfrac{dy}{dx}+y.1\right)+6y\dfrac{dy}{dx}=0$	1
		$\therefore 4x-x\dfrac{dy}{dx}-y+6y\dfrac{dy}{dx}=0$	
		$\therefore (6y-x)\dfrac{dy}{dx}=y-4x$	
		$\therefore \dfrac{dy}{dx}=\dfrac{y-4x}{6y-x}$	½
		at (3,1)	
		$\therefore \dfrac{dy}{dx}=\dfrac{1-4(3)}{6(1)-3}$	
		$\therefore \dfrac{dy}{dx}=\dfrac{-11}{3}$	½
		$\therefore$ slope of tangent , $m=\dfrac{-11}{3}$	
		Equation of tangent at $(3,1)$ is	
		$y-1=\dfrac{-11}{3}(x-3)$	½
		$\therefore 3y-3=-11x+33$	
		$\therefore 11x+3y-36=0$	½
		$\therefore$ slope of normal , $m'=\dfrac{-1}{m}=\dfrac{3}{11}$	
		Equation of normal at $(3,1)$ is	
		$y-1=\dfrac{3}{11}(x-3)$	½

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2.	e)	$\therefore 11y-11=3x-9$ $\therefore 3x-11y+2=0$ --	½
	f)	A telegraph wire hangs in the form of a curve $y=a\log\sec\left(\frac{x}{a}\right)$ where 'a' is constant. Show that the curvature at any point is $\frac{1}{a}\cos\left(\frac{x}{a}\right)$	**04**
	Ans	$y=a\log\sec\left(\frac{x}{a}\right)$	
		$\therefore \frac{dy}{dx}=a\frac{1}{\sec\left(\frac{x}{a}\right)}\sec\left(\frac{x}{a}\right)\tan\left(\frac{x}{a}\right)\frac{1}{a}$	1
		$\therefore \frac{dy}{dx}=\tan\left(\frac{x}{a}\right)$	
		$\therefore \frac{d^2y}{dx^2}=\sec^2\left(\frac{x}{a}\right)\frac{1}{a}$	1
		$\therefore$ Radius of curvature $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$	
		$=\frac{\left[1+\tan^2\left(\frac{x}{a}\right)\right]^{3/2}}{\frac{1}{a}\sec^2\left(\frac{x}{a}\right)}$	1
		$=\frac{a\left[\sec^2\left(\frac{x}{a}\right)\right]^{3/2}}{\sec^2\left(\frac{x}{a}\right)}$	
		$=\frac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	f)	$\therefore$ Radius of curvature $\rho = \dfrac{a\sec^3\left(\frac{x}{a}\right)}{\sec^2\left(\frac{x}{a}\right)}$	
		$= a\sec\left(\frac{x}{a}\right)$	1
		$\therefore$ curvature $\kappa = \dfrac{1}{\rho} = \dfrac{1}{a}\cos\left(\frac{x}{a}\right)$	
		--	
3.		**Attempt any FOUR of the following:**	**16**
	a)	Evaluate $\int_0^{\pi/2} \dfrac{dx}{1+\sqrt{\tan x}}$	**04**
	Ans	$\int_0^{\pi/2} \dfrac{dx}{1+\sqrt{\tan x}}$	
		$= \int_0^{\pi/2} \dfrac{dx}{1+\dfrac{\sqrt{\sin x}}{\sqrt{\cos x}}}$	
		$= \int_0^{\pi/2} \dfrac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$	
		Let $I = \int_0^{\pi/2} \dfrac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx ------(1)$	½
		$\therefore I = \int_0^{\pi/2} \dfrac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\, dx$	1
		$\therefore I = \int_0^{\pi/2} \dfrac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx -------(2)$	
		add (1) and (2)	
		$\therefore I + I = \int_0^{\pi/2} \dfrac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx + \int_0^{\pi/2} \dfrac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	a)	$\therefore 2I=\int_0^{\pi/2}\frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$	1
		$\therefore 2I=\int_0^{\pi/2}1\,dx$	
		$\therefore 2I=[x]_0^{\pi/2}$	½
		$\therefore 2I=\frac{\pi}{2}-0$	½
		$\therefore I=\frac{\pi}{4}$	½
		--	
	b)	Find the area bounded by the curve $y^2=9x$ and $x^2=9y$	**04**
	Ans	$y^2=9x$ ---------(1)	
		$x^2=9y$	
		$\therefore \text{eq}^n.(1)\Rightarrow \left(\frac{x^2}{9}\right)^2=9x$	
		$x^4=(9)^3x$	
		$\therefore x^4-(9)^3x=0$	
		$\therefore x\left(x^3-(9)^3\right)=0$	
		$\therefore x=0,9$	1
		Area $A=\int_a^b(y_1-y_2)dx$	
		$\therefore A=\int_0^9\left(3\sqrt{x}-\frac{x^2}{9}\right)dx$	1
		$\therefore A=\left(\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{27}\right)_0^9$	1
		$\therefore A=\left(2(9)^{\frac{3}{2}}-\frac{(9)^3}{27}\right)-0$	
		$\therefore A=27$	1
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	Solve: $\frac{dy}{dx}=(x+y)^2$	
	Ans	$\frac{dy}{dx}=(x+y)^2$ ---------(1)	
		Put $x+y=v$	
		$\therefore 1+\frac{dy}{dx}=\frac{dv}{dx}$	½
		$\therefore \frac{dy}{dx}=\frac{dv}{dx}-1$	½
		From (1) , $\frac{dv}{dx}-1=v^2$	½
		$\therefore \frac{dv}{dx}=1+v^2$	
		$\therefore \frac{dv}{1+v^2}=dx$	
		$\therefore \int\frac{dv}{1+v^2}=\int dx$	1
		$\therefore \tan^{-1}v=x+c$	1
		$\therefore \tan^{-1}(x+y)=x+c$	½
		--	**04**
	d)	Solve : $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$	
	Ans	$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$	
		Put $y=vx$	
		$\frac{dy}{dx}=v+x\frac{dv}{dx}$	1
		$\therefore v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2x(vx)}$	½
		$\therefore v+x\frac{dv}{dx}=\frac{x^2(1+v^2)}{2vx^2}$	
		$\therefore v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$	
		$\therefore x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$\therefore x\frac{dv}{dx}=\frac{1+v^2-2v^2}{2v}$	
		$\therefore x\frac{dv}{dx}=\frac{1-v^2}{2v}$	
		$\therefore \frac{2v}{1-v^2}dv=\frac{1}{x}dx$	
		$\therefore -\int\frac{-2v}{1-v^2}dv=\int\frac{1}{x}dx$	1
		$\therefore -\log\left(1-v^2\right)=\log x+c$	1
		$\therefore -\log\left(1-\frac{y^2}{x^2}\right)=\log x+c$	½
		----------	
	e)	Solve: $\left(1+x\right)\frac{dy}{dx}-y=e^{3x}\left(1+x\right)^2$	**04**
	Ans	$\left(1+x\right)\frac{dy}{dx}-y=e^{3x}\left(1+x\right)^2$	
		$\therefore \frac{dy}{dx}-\frac{y}{1+x}=e^{3x}\left(1+x\right)$	½
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=-\frac{1}{1+x},Q=e^{3x}\left(1+x\right)$	½
		$\therefore I.F.=e^{\int Pdx}=e^{-\int\frac{1}{1+x}dx}$	
		$\therefore I.F.=e^{-\log(1+x)}=\frac{1}{1+x}$	1
		Solution is	
		$y.I.F.=\int QI.F.dx+c$	
		$\therefore y\frac{1}{1+x}=\int e^{3x}\left(1+x\right)\left(\frac{1}{1+x}\right)dx+c$	1
		$\therefore \frac{y}{1+x}=\int e^{3x}dx+c$	
		$\therefore \frac{y}{1+x}=\frac{e^{3x}}{3}+c$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	f)	Evaluate : $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$	**04**
	Ans	Let $I=\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ --------------------(1)	
		$\therefore I=\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$	1
		$\therefore I=\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$	½
		$\therefore I=\int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}\,dx-\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$	
		$\therefore I=\int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}\,dx-I$	
		$\therefore 2I=\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$	½
		Put $\cos x=t$	
		$\therefore -\sin x dx=dt$	
		$\therefore \sin x dx=-dt$	½
		$x\to 0$, $t\to 1$	
		$x\to \pi$, $t\to -1$	
		$\therefore 2I=\pi\int_1^{-1} \frac{1}{1+t^2}\,(-dt)$	
		$\therefore 2I=-\pi\int_1^{-1} \frac{1}{1+t^2}\,dt$	½
		$\therefore 2I=-\pi\left(\tan^{-1} t\right)_1^{-1}$	½
		$\therefore 2I=-\pi\left(\tan^{-1}(-1)-\tan^{-1}1\right)$	
		$\therefore 2I=-\pi\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)$	
		$\therefore I=\frac{\pi^2}{4}$	½
		--	
4.		**Attempt any FOUR of the following:**	**16**

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.			
	a)	Evaluate : $\int_3^5 \frac{\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}}\,dx$	
	Ans	Let $I=\int_3^5 \frac{\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}}\,dx$ --------------------(1)	
		$I=\int_3^5 \frac{\sqrt{8-(3+5-x)}}{\sqrt{(3+5-x)}+\sqrt{8-(3+5-x)}}\,dx$	1
		$\therefore I=\int_3^5 \frac{\sqrt{8-8+x}}{\sqrt{8-x}+\sqrt{8-8+x}}\,dx$	
		$\therefore I=\int_3^5 \frac{\sqrt{x}}{\sqrt{8-x}+\sqrt{x}}\,dx$ --------------------(2)	
		add (1) and (2)	
		$I+I=\int_3^5 \frac{\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}}\,dx+\int_3^5 \frac{\sqrt{x}}{\sqrt{8-x}+\sqrt{x}}\,dx$	1
		$\therefore 2I=\int_3^5 \frac{\sqrt{8-x}+\sqrt{x}}{\sqrt{x}+\sqrt{8-x}}\,dx$	
		$\therefore 2I=\int_3^5 1\,dx$	
		$\therefore 2I=\left[x\right]_3^5$	1
		$\therefore 2I=5-3$	
		$\therefore I=1$	1
		--	
	b)	Evaluate $\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$	**04**
	Ans	Let $I=\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$	
		$I=\int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$	1
		$I=\int_0^{\frac{\pi}{4}} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)dx$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	$I=\int_0^{\frac{\pi}{4}}\log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$	½
		$I=\int_0^{\frac{\pi}{4}}\log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$	
		$I=\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{1+\tan x}\right)dx$	½
		$I=\int_0^{\frac{\pi}{4}}\left[\log 2-\log\left(1+\tan x\right)\right]dx$	
		$I=\log 2\int_0^{\frac{\pi}{4}}dx-\int_0^{\frac{\pi}{4}}\log\left(1+\tan x\right)dx$	
		$I=\log 2\left[x\right]_0^{\frac{\pi}{4}}-I$	½
		$2I=\log 2\left[\frac{\pi}{4}-0\right]$	
		$I=\frac{\pi}{8}\log 2$	1
		--	**04**
	c)	Solve D.E. $\left(2xy+y^2\right)dx+\left(x^2+2xy+\sin y\right)dy=0$	
	Ans	Let $M=2xy+y^2$ $\quad N=x^2+2xy+\sin y$	
		$\therefore\frac{\partial M}{\partial y}=2x+2y$ $\quad\therefore\frac{\partial N}{\partial x}=2x+2y$	1
		$\therefore\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$	1
		$\therefore$ D.E. exact	
		Solution is	
		$\int_{y-\text{constant}}M\ dx+\int_{\text{terms not containing 'x'}}Ndy=c$	
		$\therefore\int_{y-\text{constant}}\left(2xy+y^2\right)\ dx+\int\sin y\ dy=c$	1
		$\therefore x^2y+xy^2-\cos y=c$	1
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	Find the area enclosed between the parabola $y=x^2$ and the line $y=4$.	**04**
	Ans	$y=x^2$	
		$4=x^2$	
		$\therefore x=\pm 2$	1
		$\therefore A=\int_{-2}^{2}\left(x^2-4\right)$	1
		$A=\left(\frac{x^3}{3}-4x\right)_{-2}^{2}$	
		$A=\left(\frac{(2)^3}{3}-4(2)\right)-\left(\frac{(-2)^3}{3}-4(-2)\right)$	1
		$\therefore A=\frac{32}{3}$ or 10.667	1
		--	
	e)	Evaluate : $\int\frac{x}{\left(x^2-1\right)\left(x^2+2\right)}\,dx$	**04**
	Ans	$\int\frac{x}{\left(x^2-1\right)\left(x^2+2\right)}\,dx$	
		Put $x^2=t$	
		$\therefore 2xdx=dt$	1
		$\therefore xdx=\frac{dt}{2}$	
		$\therefore \int\frac{1}{(t-1)(t+2)}\ \frac{dt}{2}$	
		$\therefore \frac{1}{(t-1)(t+2)}=\frac{A}{t-1}+\frac{B}{t+2}$	
		$1=(t+2)A+(t-1)B$	
		Put $t=1$	
		$\therefore A=\frac{1}{3}$	1
		Put $t=-2$	
		$\therefore B=-\frac{1}{3}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$\therefore \dfrac{1}{(t-1)(t+2)} = \dfrac{\frac{1}{3}}{t-1} + \dfrac{-\frac{1}{3}}{t+2}$	
		$\therefore \dfrac{1}{2}\int \dfrac{1}{(t-1)(t+2)}\,dt = \dfrac{1}{2}\left(\dfrac{1}{3}\int \dfrac{1}{t-1}\,dt - \dfrac{1}{3}\int \dfrac{1}{t+2}\,dt\right)$	
		$= \dfrac{1}{6}\log(t-1) - \dfrac{1}{6}\log(t+2) + c$	½
		$= \dfrac{1}{6}\log(x^2-1) - \dfrac{1}{6}\log(x^2+2) + c$	½
		--	
	f)	Show that $y^2 = ax^2$ is a solution of $x\left(\dfrac{dy}{dx}\right)^2 - 2y\dfrac{dy}{dx} + ax = 0$	**04**
	Ans	$y^2 = ax^2$	
		$\therefore 2y\dfrac{dy}{dx} = 2ax$	1
		$\therefore \dfrac{dy}{dx} = \dfrac{ax}{y}$	
		$\therefore x\left(\dfrac{dy}{dx}\right)^2 - 2y\dfrac{dy}{dx} + ax$	1
		$= x\left(\dfrac{ax}{y}\right)^2 - 2y\left(\dfrac{ax}{y}\right) + ax$	
		$= \dfrac{xa^2x^2}{y^2} - 2ax + ax$	
		$= \dfrac{xa^2x^2}{ax^2} - ax$	1
		$= ax - ax$	
		$= 0$	1
		--	
5.		**Attempt any <u>FOUR</u> of the following:**	**16**
	a)	A room has 3 electric lamps. From a collection of 15 electric bulbs of which only 10 are good , 3 are selected at random and put in the lamps. Find the probability that the room is lighted by atleast one of the bulbs.	**04**
	Ans	$P = P(\text{room is lighted by atleast one of the bulbs})$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	a)	$P=1-P\left(\text{room is not lighted}\right)$	
		$P=1-\dfrac{^{5}C_3}{^{15}C_3}$	2
		$P=\dfrac{89}{91}$ or 0.9780	2
		OR	
		$P=P\left(\text{room is lighted by atleast one of the bulbs}\right)$	
		$P=\dfrac{^{10}C_1\times{}^{5}C_2+{}^{10}C_2\times{}^{5}C_1+{}^{10}C_3}{^{15}C_3}$	2
		$P=\dfrac{89}{91}$ or 0.9780	2
	b)	If the probability of bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.(Given: $e^2=7.4$)	**04**
	Ans	$p=0.001$ $\quad n=2000$	
		$\therefore m=np=0.001\times2000=2$	
		$p\left(\text{more than }2\right)=p\left(3\right)+p\left(4\right)+p\left(5\right)+...$	
		$=1-\left[p(0)+p(1)+p(2)\right]$	
		$=1-\left[\dfrac{e^{-2}\cdot\left(2\right)^0}{0!}+\dfrac{e^{-2}\cdot\left(2\right)^1}{1!}+\dfrac{e^{-2}\cdot\left(2\right)^2}{2!}\right]$	2
		$=0.3243$	2
	c)	Fit a Poisson distribution.	**04**
		<table><tr><td>x_i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f_i</td><td>12</td><td>60</td><td>15</td><td>02</td><td>01</td></tr></table>	
	Ans	$\text{Mean}=m=\dfrac{\sum f_ix_i}{\sum f_i}$	
		$\therefore m=\dfrac{1\left(12\right)+2\left(60\right)+3\left(15\right)+4\left(2\right)+5\left(1\right)}{12+60+15+2+1}$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$\therefore m=\frac{190}{90}=2.11$	2
		Poisson distribution is ,	
		$P(x=r)=\frac{e^{-m}m^r}{r!}$	
		$\therefore P(r)=\frac{e^{-2.11}(2.11)^r}{r!}$	2
		--	
	d)	Evaluate $\int\frac{dx}{5+3\cos x}$	**04**
	Ans	$\int\frac{dx}{5+3\cos x}$	
		Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$	
		$I=\int\frac{\frac{2dt}{1+t^2}}{5+3\left(\frac{1-t^2}{1+t^2}\right)}$	1
		$=\int\frac{2dt}{5(1+t^2)+3(1-t^2)}$	
		$=2\int\frac{dt}{5+5t^2+3-3t^2}$	
		$=2\int\frac{dt}{2t^2+8}$	1
		$=\int\frac{dt}{t^2+4}$	
		$=\int\frac{dt}{t^2+(2)^2}$	1
		$=\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c$	
		$=\frac{1}{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{2}\right)+c$	1
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	e)	Evaluate : $\int_0^{\pi/2} \frac{1}{1+\cot x}\, dx$	**04**
	Ans	$\int_0^{\pi/2} \frac{1}{1+\cot x}\, dx$	
		$\therefore I = \int_0^{\pi/2} \frac{1}{1+\frac{\cos x}{\sin x}}\, dx$	
		$\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\, dx$ --------------------(1)	
		$\therefore I = \int_0^{\pi/2} \frac{\sin\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)+\cos\left(\pi/2-x\right)}\, dx$	1
		$\therefore I = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\, dx$ --------------------(2)	
		add (1) and (2)	
		$\therefore I+I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\, dx + \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\, dx$	1
		$2I = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x}\, dx$	
		$2I = \int_0^{\pi/2} 1\, dx$	1
		$2I = \left[x\right]_0^{\pi/2}$	
		$2I = \frac{\pi}{2}-0$	
		$\therefore I = \frac{\pi}{4}$	1
		--	
	f)	Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$	**04**
	Ans	$\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	f)	$\frac{dy}{dx}=e^{-2y}\left(e^{3x}+x^2\right)$ $\int e^{2y}dy=\int\left(e^{3x}+x^2\right)dx$ $\frac{e^{2y}}{2}=\frac{e^{3x}}{3}+\frac{x^3}{3}+c$	2 2
6.		**Attempt any FOUR of the following:**	**16**
	a)	In a sample of 1000 cases . the mean of certain test is 14 and satandard deviation is 2.5. Assuming the distribution to be normal find: (i) How many students score between 12 and 15 ? (ii) How many students score above 18 ? Given $\text{A}(0.8)=0.2881$ $\text{A}(0.4)=0.1554$ $\text{A}(1.6)=0.4452$	**04**
	Ans	Given $\bar{x}=14$ $\sigma=2.5$ $N=1000$ $i)$ For $x=12$ $z=\frac{x-\bar{x}}{\sigma}=\frac{12-14}{2.5}=-0.8$ For $x=12$ $z=\frac{x-\bar{x}}{\sigma}=\frac{15-14}{2.5}=0.4$ $\therefore\ p(\text{between 12 and 15})=A(-0.8)+A(0.4)$ $=0.2881+0.1554$ $=0.4435$ $\therefore$ No.of students $=N\cdot p$ $=1000\times0.4435$ $=443.5\approx444$ $ii)$ For $x=18$ $z=\frac{x-\bar{x}}{\sigma}=\frac{18-14}{2.5}=1.6$ $\therefore\ p(\text{above 18})=0.5-A(1.6)$ $=0.5-0.4452$ $=0.0548$	½ 1 ½ ½ 1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)	$\therefore$ No. of students $= N \cdot p = 1000 \times 0.0548$ $= 54.8 \approx 55$	½
	b)	If $P(A)=\frac{1}{5}, P(B')=\frac{3}{5}$ and $P\left(\frac{A}{B}\right)=\frac{3}{4}$. Find $P(A\cap B)$ and $P\left(\frac{B}{A}\right)$	**04**
	Ans	Given $P(A)=\frac{1}{5}, P(B')=\frac{3}{5}$ and $P\left(\frac{A}{B}\right)=\frac{1}{4}$	
		$P(B)=1-P(B')=1-\frac{3}{5}=\frac{2}{5}$	1
		$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$ $\therefore \frac{3}{4}=\frac{P(A\cap B)}{\frac{2}{5}}$ $\therefore \frac{3}{4}\times\frac{2}{5}=P(A\cap B)$ $\therefore P(A\cap B)=\frac{3}{10}$	1
		and $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$ $\therefore P\left(\frac{B}{A}\right)=\frac{\frac{3}{10}}{\frac{1}{5}}$	1
		$\therefore P\left(\frac{B}{A}\right)=\frac{3}{2}$	1
	c)	A metal wire of 36 cm long is bent to form a rectangle.Find its dimensions when its area is maximum.	**04**
	Ans	Let length of rectangle = x , breadth = y $2x+2y=36$ $\therefore x+y=18 \qquad \therefore y=18-x$	1
		Area $A=xy$ $\therefore A=x(18-x)$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$\therefore A = 18x - x^2$	
		$\therefore \frac{dA}{dx} = 18 - 2x$	½
		$\therefore \frac{d^2A}{dx^2} = -2$	½
		Consider $\frac{dA}{dx} = 0$	½
		$18 - 2x = 0$	
		$\therefore x = 9$	
		at $x = 9$ $\therefore \frac{d^2A}{dx^2} = -2 < 0$	
		$\therefore$ Aera is maximum at $x = 9$	
		$\therefore x = 9$	½
		$\therefore y = 9$	
		--	
	d)	Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$	**04**
	Ans	$y^2 = 4ax$ ---------(1)	
		$x^2 = 4ay$	
		$\therefore y = \frac{x^2}{4a}$	
		$\therefore \text{eq}^n.(1) \Rightarrow$	
		$\left(\frac{x^2}{4a}\right)^2 = 4ax$	
		$\frac{x^4}{16a^2} = 4ax$	
		$\therefore x^4 = 64a^3x$	
		$\therefore x^4 - 64a^3x = 0$	
		$\therefore x\left(x^3 - 64a^3\right) = 0$	1
		$\therefore x = 0, 4a$	
		Area $A = \int_a^b \left(y_1 - y_2\right) dx$	
		$\therefore A = \int_0^4 \left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4a}\right) dx$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	d)	$\therefore A=\int_0^{4a}\left(2\sqrt{a}x^{\frac{1}{2}}-\frac{x^2}{4a}\right)dx$	
		$\therefore A=\left(\frac{2\sqrt{a}x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12a}\right)_0^{4a}$	1
		$\therefore A=\left(\frac{2\sqrt{a}(4a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(4a)^3}{12a}\right)-0$	
		$\therefore A=\frac{16a^2}{3}$ or $5.333a^2$	1
		--	
	e)	Find the equation of tangent to the curve $y=9x^2-12x+7$ which is parallel to $x-$axis.	**04**
	Ans	$y=9x^2-12x+7$	
		$\therefore \frac{dy}{dx}=18x-12$	1
		Slope of tangent which is parallel to $x-axis$ is $m=0$	½
		$\therefore 18x-12=0$	
		$\therefore x=\frac{12}{18}=\frac{2}{3}$	½
		$\therefore y=9\left(\frac{2}{3}\right)^2-12\left(\frac{2}{3}\right)+7=3$	½
		$\therefore (x,y)=\left(\frac{2}{3},3\right)$	
		at $\left(\frac{2}{3},3\right)$	
		Equation of tangent is,	
		$\therefore y-y_1=m(x-x_1)$	
		$y-3=0\left(x-\frac{2}{3}\right)$	1
		$y=3$	½
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	f)	The probability that a man aged 65 will live to 75 is 0.65. What is the probability that out of 10 men which are now 65, 7 will live to 75?	**04**
	Ans	Given	
		$p=0.65$, $q=1-0.65=0.35$	1
		$n=10$, $r=7$	
		$\because p(r)={}^{n}C_{r}(p)^{r}(q)^{n-r}$	
		$\therefore p(7)={}^{10}C_{7}(0.65)^{7}(0.35)^{10-7}$	2
		$\therefore p(7)=0.2522$	1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.