MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
(ISO/IEC - 27001 - 2013 Certified)

WINTER– 2019 EXAMINATION

Subject Name: Engineering Mathematics **Model Answer** **Subject Code: 17216**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any TEN of the following:**	**20**
	a)	If $z=1-3i$ Find z^2+2z+4	**02**
	Ans	z^2-2z+4	½
		$=(1-3i)^2+2(1-3i)+4$	½
		$=1-6i+9i^2+2-6i+4$	½
		$=1-6i+9(-1)+2-6i+4$	½
		$=-2-12i$	
		--	
	b)	Find modulus and amplitude of $\frac{1}{2}+i\frac{\sqrt{3}}{2}$	**02**
	Ans	Let $z=\frac{1}{2}+i\frac{\sqrt{3}}{2}$	
		$x=\frac{1}{2}, y=\frac{\sqrt{3}}{2}$	
		$r=\sqrt{x^2+y^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$	1
		$\therefore r=1$	
		$\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b)	$\theta = \tan^{-1}\left(\sqrt{3}\right) = 60^0 \ or \ \frac{\pi}{3}$	1
	c)	State whether the function $f(x) = \frac{e^x + e^{-x}}{2}$ is even or odd.	**02**
	Ans	$f(x) = \frac{e^x + e^{-x}}{2}$	
		$\therefore f(-x) = \frac{e^{-x} + e^{-(-x)}}{2}$	1
		$\therefore f(-x) = \frac{e^{-x} + e^{x}}{2}$	½
		$\therefore f(-x) = f(x)$	½
		$\therefore$ function is even	
	d)	If $f\left(x\right) = 3x^2 - 5x + 7$ show that $f\left(-1\right) = 3f\left(1\right)$	**02**
	Ans	$f\left(-1\right) = 15$	1
		$f\left(1\right) = 5$	½
		$\therefore 3f\left(1\right) = 15$	½
		$\therefore f\left(-1\right) = 3f\left(1\right)$	
	e)	Evaluate $\lim\limits_{x \to 2} \frac{x^2 - 4}{x - 2}$	**02**
	Ans	$\lim\limits_{x \to 2} \frac{x^2 - 4}{x - 2}$	
		$= \lim\limits_{x \to 2} \frac{\left(x-2\right)\left(x+2\right)}{x - 2}$	1
		$= \lim\limits_{x \to 1} \left(x + 2\right)$	
		$= 2 + 2$	1
		$= 4$	
	f)	Evaluate $\lim\limits_{x \to 0} \frac{\sin 2x}{\sin 3x}$	**02**
	Ans	$\lim\limits_{x \to 0} \frac{\sin 2x}{\sin 3x}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f)	$=\lim_{x\to 0}\dfrac{\dfrac{\sin 2x}{2x}\times 2}{\dfrac{\sin 3x}{3x}\times 3}$	1
		$=\dfrac{2\left(\lim_{x\to 0}\dfrac{\sin 2x}{2x}\right)}{3\left(\lim_{x\to 0}\dfrac{\sin 3x}{3x}\right)}$	½
		$=\dfrac{2(1)}{3(1)}$	
		$=\dfrac{2}{3}$	½
	g)	Evaluate $\lim_{x\to\infty}\left[1+\dfrac{2}{x}\right]^x$	**02**
	Ans	$\lim_{x\to\infty}\left[1+\dfrac{2}{x}\right]^x$	
		$=\lim_{x\to\infty}\left[\left[1+\dfrac{2}{x}\right]^{\frac{x}{2}}\right]^2$	1
		$=e^2$	1
	h)	Find $\dfrac{dy}{dx}$ if $y=\sin(\log x)+\cos(\log x)$	**02**
	Ans	$y=\sin(\log x)+\cos(\log x)$	
		$\therefore\dfrac{dy}{dx}=\cos(\log x)\dfrac{1}{x}-\sin(\log x)\dfrac{1}{x}$	2
		$\therefore\dfrac{dy}{dx}=\dfrac{\cos(\log x)-\sin(\log x)}{x}$	
	i)	If $x^2+y^2=25$ find $\dfrac{dy}{dx}$	**02**
	Ans	$x^2+y^2=25$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
1.	i)	$\therefore 2x+2y\frac{dy}{dx}=0$	1½
		$\therefore \frac{dy}{dx}=\frac{-2x}{2y}$	
		$\therefore \frac{dy}{dx}=\frac{-x}{y}$	½
	j)	If $x=at^2$ and $y=2at$ find $\frac{dy}{dx}$	**02**
	Ans	$x=at^2$	
		$\therefore \frac{dx}{dt}=2at$	½
		$y=2at$	
		$\therefore \frac{dy}{dt}=2a$	½
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2a}{2at}$	
		$\therefore \frac{dy}{dx}=\frac{1}{t}$	1
	k)	Show that root of equation $x^2+x-3=0$ lies between 2 and 3.	**02**
	Ans	$f(x)=x^2+x-3$	
		$f(2)=2^2+2-3=3>0$	1
		$f(3)=3^2+3-3=9>0$	1
		root not in 2 and 3	
	l)	Find the first iteration by Gauss seidal method.	**02**
		$10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$	
	Ans	$x=\frac{13-y-2z}{10}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	l)	$y=\frac{14-3x-z}{10}$	1
		$z=\frac{15-2x-3y}{10}$	
		Initial approximations : $y_0=z_0=0$	
		$x==\frac{13-0-2(0)}{10}=1.3$	
		$y=\frac{14-3(1.3)-0}{10}=1.01$	1
		$z=\frac{15-2(1.3)-3(1.01)}{10}=0.937$	
		$x=1.3$	
		$y=1.01$	
		$z=0.937$	

2.		**Solve any FOUR of the following:**	**16**
	a)	Express in polar form $z=-1+\sqrt{3}\ i$	**04**
	Ans	Let $z=-1+\sqrt{3}\ i$	
		$x=-1\ ,\ y=\sqrt{3}$	
		$r=\sqrt{x^2+y^2}=\sqrt{(-1)^2+\left(\sqrt{3}\ \right)^2}=\sqrt{4}$	1
		$\therefore r=2$	
		$\theta=\pi-\tan^{-1}\left(\frac{y}{x}\right)$	
		$\theta=\pi-\tan^{-1}\left(\left\|\frac{\sqrt{3}}{-1}\right\|\right)$	1
		$\theta=\pi-\frac{\pi}{3}$	
		$\theta=\frac{2\pi}{3}$	1
		In polar form, $z=r(\cos\theta+i\sin\theta)$	
		$\therefore -1+\sqrt{3}\ i=2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	b)	Simplify using De-Movier's Theorem ,	**04**
		$\dfrac{(\cos 2\theta + i\sin 2\theta)(\cos\theta - i\sin\theta)^4}{(\cos 3\theta + i\sin 3\theta)(\cos 5\theta - i\sin 5\theta)^3}$	
	Ans	$\dfrac{(\cos 2\theta + i\sin 2\theta)(\cos\theta - i\sin\theta)^4}{(\cos 3\theta + i\sin 3\theta)(\cos 5\theta - i\sin 5\theta)^3}$	
		$= \dfrac{(\cos\theta + i\sin\theta)^2(\cos\theta + i\sin\theta)^{-4}}{(\cos\theta + i\sin\theta)^3(\cos\theta + i\sin\theta)^{-15}}$	1
		$= (\cos\theta + i\sin\theta)^{2-4-3+15}$	1
		$= (\cos\theta + i\sin\theta)^{10}$	1
		$= \cos 10\theta + i\sin 10\theta$	1
	c)	Using Euler's formula prove that $\cosh^2\theta - \sinh^2\theta = 1$	**04**
	Ans	$\cosh^2\theta - \sinh^2\theta$	
		$= \left(\dfrac{e^{\theta}+e^{-\theta}}{2}\right)^2 - \left(\dfrac{e^{\theta}-e^{-\theta}}{2}\right)^2$	½
		$= \dfrac{1}{4}(e^{\theta}+e^{-\theta})^2 - \dfrac{1}{4}(e^{\theta}-e^{-\theta})^2$	½
		$= \dfrac{1}{4}(e^{2\theta}+2e^{\theta}e^{-\theta}+e^{-2\theta}) - \dfrac{1}{4}(e^{2\theta}-2e^{\theta}e^{-\theta}+e^{-2\theta})$	1
		$= \dfrac{1}{4}(4e^{\theta}e^{-\theta}) = \dfrac{1}{4}(4e^{0})$	1
		$=1$	1
	d)	Find cube root and unity by De Moivre's theorem	**04**
	Ans	Let $x = \sqrt[3]{1}$ $\quad \therefore x^3 = 1$	
		Put $x^3 = z \therefore x = z^{\frac{1}{3}}$	
		$\therefore z = 1 + 0i$	
		$\text{Re}(z) = 1, \text{Im}(z) = 0$	
		$r = \lvert z\rvert = \sqrt{1+0} = 1$	½
		$\theta = \tan^{-1}\left(\dfrac{0}{1}\right) = 0$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$z = r(\cos\theta + i\sin\theta)$	
		$z = 1(\cos 0 + i\sin 0)$	½
		In general polar form , $z = r\left(\cos(2\pi k+\theta) + i\sin(2\pi k+\theta)\right)$	
		$z = 1(\cos 2\pi k + i\sin 2\pi k)$	½
		$z^{\frac{1}{3}} = (\cos 2\pi k + i\sin 2\pi k)^{\frac{1}{3}}$	
		$z^{\frac{1}{3}} = \cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right) \quad ; \quad k = 0,1,2$	½
		when $k = 0$	
		$z_1 = \cos 0 + i\sin 0 = 1$	½
		when $k = 1$	
		$z_2 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$	½
		when $k = 2$	
		$z_3 = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$	½
		---	
	e)	If $f(x) = \log(1+\tan x)$ show that $f\left(\frac{\pi}{4} - x\right) = \log 2 - f(x)$	**04**
	Ans	$f\left(\frac{\pi}{4} - x\right) = \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right)$	½
		$= \log\left(1 + \frac{\tan\left(\frac{\pi}{4}\right) - \tan x}{1 + \tan\left(\frac{\pi}{4}\right)\tan x}\right)$	1
		$= \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right)$	½
		$= \log\left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x}\right)$	½
		$= \log\left(\frac{2}{1 + \tan x}\right)$	
		$= \log 2 - \log(1 + \tan x)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	e)	$=\log 2 - f(x)$	½
		--	
	f)	If $y = f(x) = \dfrac{2x-3}{3x-2}$ then show that $f(y) = x$	**04**
	Ans	$f(x) = \dfrac{2x-3}{3x-2}$	
		$\therefore f(y) = \dfrac{2y-3}{3y-2}$	
		$= \dfrac{2\left(\dfrac{2x-3}{3x-2}\right)-3}{3\left(\dfrac{2x-3}{3x-2}\right)-2}$	1
		$= \dfrac{\dfrac{2(2x-3)-3(3x-2)}{3x-2}}{\dfrac{3(2x-3)-2(3x-2)}{3x-2}}$	1
		$= \dfrac{4x-6-9x+6}{6x-9-6x+4}$	1
		$= \dfrac{-5x}{-5}$	½
		$= x$	½
		$\therefore f(y) = x$	
		--	
3.		**Solve any <u>FOUR</u> of the following:**	**16**
	a)	If $f(t) = 50\sin(100\pi t + 0.04)$ show that $f\left[\dfrac{2}{100}+t\right] = f(t)$	**04**
	Ans	$f\left(\dfrac{2}{100}+t\right) = 50\sin\left(100\pi\left(\dfrac{2}{100}+t\right)+0.04\right)$	1
		$= 50\sin(2\pi + 100\pi t + 0.04)$	1
		$= 50\sin(100\pi t + 0.04)$	1
		$= f(t)$	1
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3.	b)	If $f(x)=\log x$ show that :	**04**
		(i) $f(mn)=f(m)+f(n)$	
		(ii) $f\left(\frac{m}{n}\right)=f(m)-f(n)$	
	Ans	$f(x)=\log x$	
		$(i)\ f(mn)$	½
		$=\log(mn)$	1
		$=\log m+\log n$	½
		$=f(m)+f(n)$	
		(ii) $f\left(\frac{m}{n}\right)$	½
		$=\log\left(\frac{m}{n}\right)$	1
		$=\log m-\log n$	½
		$=f(m)-f(n)$	
		--	**04**
	c)	Evaluate $\lim_{x\to 0}\frac{10^x-2^x-5^x+1}{x^2}$	
	Ans	$\lim_{x\to 0}\frac{10^x-2^x-5^x+1}{x^2}$	
		$=\lim_{x\to 0}\frac{5^x2^x-5^x-2^x+1}{x^2}$	1
		$=\lim_{x\to 0}\frac{5^x\left(2^x-1\right)-\left(2^x-1\right)}{x^2}$	1
		$=\lim_{x\to 0}\frac{(5^x-1)\left(2^x-1\right)}{x^2}$	
		$=\lim_{x\to 0}\frac{(5^x-1)\left(2^x-1\right)}{x^2}$	1
		$=\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)$	
		$=(\log 5)(\log 2)$	1
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	Evaluate: $\lim_{x\to 0}\frac{x\tan x}{1-\cos x}$	**04**
	Ans	$\lim_{x\to 0}\frac{x\tan x}{1-\cos x}$	
		$=\lim_{x\to 0}\frac{x\tan x}{2\sin^2\left(\frac{x}{2}\right)}$	1
		$=\frac{1}{2}\lim_{x\to 0}\frac{x\tan x}{\sin^2\left(\frac{x}{2}\right)}$	
		$=\frac{1}{2}\lim_{x\to 0}\frac{\frac{x\tan x}{x^2}}{\frac{\sin^2\left(\frac{x}{2}\right)}{x^2}}$	1
		$=\frac{1}{2}\frac{\left(\lim_{x\to 0}\frac{\tan x}{x}\right)}{\left(\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\times\frac{1}{2}\right)^2}$	1
		$=2$	1
		--	
	e)	Evaluate: $\lim_{x\to\infty}\left[\sqrt{x^2+x+1}-x\right]$	**04**
	Ans	$\lim_{x\to\infty}\left[\sqrt{x^2+x+1}-x\right]$	
		$=\lim_{x\to\infty}\left[\sqrt{x^2+x+1}-x\right]\times\frac{\left[\sqrt{x^2+x+1}+x\right]}{\left[\sqrt{x^2+x+1}+x\right]}$	½
		$=\lim_{x\to\infty}\frac{\left[\left(\sqrt{x^2+x+1}\right)^2-x^2\right]}{\left[\sqrt{x^2+x+1}+x\right]}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	e)	$=\lim_{x\to\infty}\frac{\left[\left(\sqrt{x^2+x+1}\right)^2-x^2\right]}{\left[\sqrt{x^2+x+1}+x\right]}$	
		$=\lim_{x\to\infty}\frac{\left[x^2+x+1-x^2\right]}{\left[\sqrt{x^2+x+1}+x\right]}$	
		$=\lim_{x\to\infty}\frac{[x+1]}{\left[\sqrt{x^2+x+1}+x\right]}$	
		$=\lim_{x\to\infty}\frac{x\left[1+\frac{1}{x}\right]}{x\left[\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right]}$	1
		$=\lim_{x\to\infty}\frac{\left[1+\frac{1}{x}\right]}{\left[\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right]}$	
		$=\frac{[1+0]}{\left[\sqrt{1+0+0}+1\right]}$	1
		$=\frac{1}{2}$	½
		--	
	f)	Evaluate : $\lim_{x\to0}\frac{\log10+\log(x+0.1)}{x}$	**04**
	Ans	$\lim_{x\to0}\frac{\log10+\log(x+0.1)}{x}$	
		$=\lim_{x\to0}\frac{\log\left[10(x+0.1)\right]}{x}$	½
		$=\lim_{x\to0}\frac{\log(10x+1)}{x}$	
		$=\lim_{x\to0}\log(10x+1)^{\frac{1}{x}}$	½
		$=\lim_{x\to0}\left[\log(1+10x)^{\frac{1}{x}}\right]$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	f)	$=\lim_{x\to 0}\left[\log(1+10x)^{\frac{1}{10x}}\right]^{10}$	1
		$=\log e^{10}$	1
		$=10\log e=10(1)$	
		$=10$	1
4.		**Solve any FOUR of the following:**	**16**
	a)	Using first principle of derivatives find derivatives of $f(x)=\sin x$	**04**
	Ans	$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	
		$\therefore \frac{dy}{dx}=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$	1
		$\therefore \frac{dy}{dx}=\lim_{h\to 0}\frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$	1
		$\therefore \frac{dy}{dx}=\lim_{h\to 0}\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$	
		$\therefore \frac{dy}{dx}=2\lim_{h\to 0}\cos\left(\frac{2x+h}{2}\right)\left(\frac{\lim_{h\to 0}\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\times\frac{1}{2}\right)$	
		$\therefore \frac{dy}{dx}=2\cos\left(\frac{2x+0}{2}\right)\left(1\times\frac{1}{2}\right)$	1
		$\therefore \frac{dy}{dx}=\cos x$	1
	b)	If u and v are differentiable functions of x then prove that $\frac{d}{dx}uv=u\frac{dv}{dx}+v\frac{du}{dx}$	**04**
	Ans	Let $y=uv$	
		Let $\delta u,\delta v,\delta y$ are small increments in u,v,y respectively corresponding to increment δx in x.	
		$\therefore y+\delta y=(u+\delta u)(v+\delta v)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	$y+\delta y=uv+u\delta v+v\delta u+\delta u\delta v$	
		$\delta y=uv+u\delta v+v\delta u+\delta u\delta v-y$	
		$\delta y=uv+u\delta v+v\delta u+\delta u\delta v-uv$	
		$\delta y=u\delta v+v\delta u+\delta u\delta v$	1
		$\because \delta u,\delta v$ are very small.	
		$\therefore \delta u\delta v$ is negligible.	
		$\therefore \delta y=u\delta v+v\delta u$	
		$\therefore \frac{\delta y}{\delta x}=\frac{u\delta v+v\delta u}{\delta x}$	1
		$\lim\limits_{\delta x\to 0}\frac{\delta y}{\delta x}=u\lim\limits_{\delta x\to 0}\frac{\delta v}{\delta x}+v\lim\limits_{\delta x\to 0}\frac{\delta u}{\delta x}$	
		$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$	1
		--	**04**
	c)	If $x^3+y^3=4xy$ find $\frac{dy}{dx}$	
	Ans	$3x^2+3y^2\frac{dy}{dx}=4\left(x\frac{dy}{dx}+y\right)$	2
		$3x^2+3y^2\frac{dy}{dx}=4x\frac{dy}{dx}+4y$	
		$3y^2\frac{dy}{dx}-4x\frac{dy}{dx}=4y-3x^2$	1
		$\left(3y^2-4x\right)\frac{dy}{dx}=4y-3x^2$	
		$\frac{dy}{dx}=\frac{4y-3x^2}{3y^2-4x}$	1
		--	**04**
	d)	Differentiate w.r.t. x: $\tan^{-1}\left[\frac{5x}{1-6x^2}\right]$	
	Ans	$\tan^{-1}\left[\frac{5x}{1-6x^2}\right]$	
		$=\tan^{-1}\left[\frac{3x+2x}{1-(3x)(2x)}\right]$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$= \tan^{-1}[3x] + \tan^{-1}[2x]$	1
		$= \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) + \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$	
		$= \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$	2
	e)	If $y = \left(\sin^{-1}x\right)^x$ find $\frac{dy}{dx}$	**04**
	Ans	$y = \left(\sin^{-1}x\right)^x$	
		$\log y = \log\left(\sin^{-1}x\right)^x$	1
		$\log y = x\log\left(\sin^{-1}x\right)$	1
		$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{\sin^{-1}x}\frac{1}{\sqrt{1-x^2}} + \log\left(\sin^{-1}x\right)(1)$	1
		$\frac{dy}{dx} = y\left[\frac{x}{\sin^{-1}x}\frac{1}{\sqrt{1-x^2}} + \log\left(\sin^{-1}x\right)\right]$	1
		$\frac{dy}{dx} = \left(\sin^{-1}x\right)^x\left[\frac{x}{\sin^{-1}x}\frac{1}{\sqrt{1-x^2}} + \log\left(\sin^{-1}x\right)\right]$	
	f)	If $x = a\cos^3\theta$, $y = a\sin^3\theta$ find $\frac{dy}{dx}$	**04**
	Ans	$x = a\cos^3\theta$	
		$\therefore \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$	1
		$y = b\sin^3\theta$	
		$\therefore \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$	1
		$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	1
		$\therefore \frac{dy}{dx} = -\frac{a\sin\theta}{a\cos\theta} = -\tan\theta$	1

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5.		**Solve any FOUR of the following:**	**16**
	a)	Evaluate: $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$	**04**
	Ans	$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$	
		$=\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$	½
		$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	1
		$=\lim_{x\to 0}\frac{(1+x)-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	
		$=\lim_{x\to 0}\frac{1+x-1+x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	
		$=\lim_{x\to 0}\frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	
		$=\lim_{x\to 0}\frac{2}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$	½
		$=\frac{2}{\left(\sqrt{1+0}+\sqrt{1-0}\right)}$	1
		$=\frac{2}{2}$ $=1$	1
		--	
	b)	Evaluate: $\lim_{x\to\infty}\left[\frac{x+1}{x-1}\right]^x$	**04**
	Ans	$\lim_{x\to\infty}\left[\frac{x+1}{x-1}\right]^x$	
		$=\lim_{x\to\infty}\left[\frac{\frac{x+1}{x}}{\frac{x-1}{x}}\right]^x$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b)	$=\lim_{x\to\infty}\left[\frac{\left(1+\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)}\right]^{x}$	
		$=\frac{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}}{\left[\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{-x}\right]^{-1}}$	1
		$=\frac{e}{e^{-1}}$	1
		$=e^{2}$	1
	c)	Using Bisection method find the approximate root of the equation $x^3-6x+3=0$ (three iterations only)	**04**
	Ans	$x^3-6x+3=0$ $f(x)=x^3-6x+3$ $f(0)=3>0$ $f(1)=-2<0$	1
		root is in $(0,1)$ $\therefore x_1=\frac{0+1}{2}=0.5$	1
		$\therefore f(0.5)=0.125>0$ $\therefore$ root is in $(0.5,1)$ $\therefore x_2=\frac{0.5+1}{2}=0.75$	1
		$\therefore f(0.75)=-1.078<0$ $\therefore$ root is in $(0.5,0.75)$ $\therefore x_3=\frac{0.75+0.5}{2}=0.625$	1
		OR	
		$x^3-6x+3=0$ $f(x)=x^3-6x+3$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$f(0)=3>0$	
		$f(1)=-2<0$	1
		root is in $(0,1)$	

a	b	$x=\frac{a+b}{2}$	$f(x)$
0	1	0.5	0.125
0.5	1	0.75	–1.078
0.5	0.75	0.625	------

Marking scheme: 1+1+1

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d) Using Regula Falsi method find the root of equation $x^3-9x+1=0$ [Three iterations only] **04**

Ans

Let $f(x)=x^3-9x+1$

$f(0)=1>0$

$f(1)=-7<0$ 1

$\therefore$ root lies in $(1,0)$

$$x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{1(1)-0(-7)}{1-(-7)}=0.125$$ 1

$f(x_1)=-0.123<0$

$\therefore$ the root is in $(0.125,0)$

$$x_2=\frac{0.125(1)-0(-0.123)}{1-(-0.123)}=0.111$$ 1

$f(x_2)=0.002>0$

$\therefore$ the root is in $(0.111,0.125)$

$$x_3=\frac{0.111(-0.123)-0.125(0.002)}{-0.123-0.002}=0.111$$ 1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	e)	Solve the equation $x^3 - x - 1 = 0$ using Newton-Raphson method taking initial root '1' $\left[\text{Three iterations only}\right]$	
	Ans	Let $f(x) = x^3 - x - 1$ $f(1) = -1 < 0$ $f(2) = 5 > 0$ $f'(x) = 3x^2 - 1$ Initial root $x_0 = 1$ $\therefore f'(1) = 2$	1
		$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1 - \dfrac{f(1)}{f'(1)} = 1.5$	1
		$x_2 = 1.5 - \dfrac{f(1.5)}{f'(1.5)} = 1.348$	1
		$x_3 = 1.348 - \dfrac{f(1.348)}{f'(1.348)} = 1.325$	1
		OR	
		Let $f(x) = x^3 - x - 1$ $f(1) = -1 < 0$ $f(2) = 5 > 0$ $f'(x) = 3x^2 - 1$ Initial root $x_0 = 1$	1
		$x_i = x - \dfrac{f(x)}{f'(x)} = x - \dfrac{x^3 - x - 1}{3x^2 - 1}$ $= \dfrac{2x^3 + 1}{3x^2 - 1}$	1
		$x_1 = 1.5$	1
		$x_2 = 1.348$	½
		$x_3 = 1.325$	½
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	f)	Find the root of equation $x\log_e x = 1.2$ by using Bisection method.[Three iterations only]	04
	Ans	$x\log_e x = 1.2$ $x\log_e x - 1.2 = 0$ $f(x) = x\log_e x - 1.2$ $f(1) = -1.2$ $f(2) = 0.186$	1
		$\therefore$ the root is in $(1,2)$ $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$	1
		$f(1.5) = -0.592 < 0$ $\therefore$ root lies in $(1.5,2)$ $x_2 = \frac{x_1+b}{2} = \frac{1.5+2}{2} = 1.75$	1
		$f(1.75) = -0.221 < 0$ the root is in $(1.75,2)$ $x_3 = \frac{x_2+b}{2} = \frac{1.75+2}{2} = 1.875$	1
		OR Let $f(x) = x\log_e x - 1.2$ $f(1) = -1.2$ $f(2) = 0.186$ $\therefore$ the root is in $(1,2)$	1

Iteration	a	b	$x = \frac{a+b}{2}$	$f(x)$
I	1	2	1.5	-0.592
II	1.5	2	1.75	-0.221
III	1.75	2	1.875	----

Marking Scheme: 1+1+1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.		**Solve any FOUR of the following:**	**16**
	a)	Differentiate $\log\left(1+x^2\right)$ w.r.t. $\tan^{-1}x$	**04**
	Ans	Let $u=\log\left(1+x^2\right)$ and $v=\tan^{-1}x$	
		$u=\log\left(1+x^2\right)$	
		$\therefore \dfrac{du}{dx}=\dfrac{1}{1+x^2}\dfrac{d}{dx}\left(1+x^2\right)$	
		$\therefore \dfrac{du}{dx}=\dfrac{2x}{1+x^2}$	1
		$v=\tan^{-1}x$	
		$\therefore \dfrac{dv}{dx}=\dfrac{1}{1+x^2}$	1
		$\therefore \dfrac{du}{dv}=\dfrac{\frac{2x}{1+x^2}}{\frac{1}{1+x^2}}$	1
		$\therefore \dfrac{du}{dv}=2x$	1
		--	
	b)	If $y=e^{\tan^{-1}x}$ show that $\left(1+x^2\right)\dfrac{d^2y}{dx^2}+\left(2x-1\right)\dfrac{dy}{dx}=0$	**04**
	Ans	$y=e^{\tan^{-1}x}$	
		$\therefore \dfrac{dy}{dx}=e^{\tan^{-1}x}\dfrac{1}{1+x^2}$	1
		$\therefore \left(1+x^2\right)\dfrac{dy}{dx}=y$	
		$\therefore \left(1+x^2\right)\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}\left(2x\right)=\dfrac{dy}{dx}$	1
		$\therefore \left(1+x^2\right)\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}\left(2x\right)-\dfrac{dy}{dx}=0$	1
		$\therefore \left(1+x^2\right)\dfrac{d^2y}{dx^2}+\left(2x-1\right)\dfrac{dy}{dx}=0$	1
		OR	
		$y=e^{\tan^{-1}x}$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore \frac{dy}{dx} = e^{\tan^{-1}x}\frac{1}{1+x^2}$	1
		$\therefore \frac{dy}{dx} = \frac{y}{1+x^2}$	
		$\therefore \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\frac{y}{1+x^2} - y\left(2x\right)}{\left(1+x^2\right)^2}$	
		$\therefore \frac{d^2y}{dx^2} = \frac{y - y\left(2x\right)}{\left(1+x^2\right)^2}$	
		$\therefore \frac{d^2y}{dx^2} = \frac{y\left(1-2x\right)}{\left(1+x^2\right)^2}$	1
		$L.H.S. = \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx}$	
		$= \left(1+x^2\right)\frac{y\left(1-2x\right)}{\left(1+x^2\right)^2} + \left(2x-1\right)\frac{y}{1+x^2}$	1
		$= -\left(2x-1\right)\frac{y}{1+x^2} + \left(2x-1\right)\frac{y}{1+x^2}$	1
		$= 0$	
		$= R.H.S.$	
	c)	Using Gauss-elimination method	**04**
		$2x+y+z=10$; $3x+2y+3z=18$; $x+4y+9z=16$	
	Ans	$4x+2y+2z=20$ and $8x+4y+4z=40$	1
		$3x+2y+3z=18$ and $x+4y+9z=16$	
		$-$ ______ $-$ ______	
		$x-z=2$ $7x-5z=24$	
		$\therefore 5x-5z=10$	
		$7x-5z=24$	
		$-$ ______	
		$-2x=-14$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$\therefore x = 7$	1
		$y = -9$	1
		$z = 5$	1
	d)	Solve by Jacobi's method	**04**
		$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ (upto three iterations only)	
	Ans	$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$	
		$x = \frac{1}{20}(17 - y + 2z)$	
		$y = \frac{1}{20}(-18 - 3x + z)$	
		$z = \frac{1}{20}(25 - 2x + 3y)$	1
		Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 0.85$	
		$y_1 = -0.9$	1
		$z_1 = 1.25$	
		$x_2 = 1.02$	
		$y_2 = -0.965$	1
		$z_2 = 1.03$	
		$x_3 = 1.001$	
		$y_3 = -1.001$	
		$z_3 = 1.003$	1
	e)	Solve the following equations Jacobi's method	**04**
	Ans	$2x + 3y - 4z = 1$, $5x + 9y + 3z = 17$, $8x - 2y - z = 5$ (up to three iterations)	
		$x = \frac{1}{8}(5 + 2y + z)$	
		$y = \frac{1}{9}(17 - 5x - 3z)$	
		$z = -\frac{1}{4}(1 - 2x - 3y)$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	e)	Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 0.625$ $y_1 = 1.889$ $z_1 = -0.25$	1
		$x_2 = 1.066$ $y_2 = 1.625$ $z_2 = 1.479$	1
		$x_3 = 1.216$ $y_3 = 0.804$ $z_3 = 1.502$	1
		--	
	f)	With the following system of equations $5x - y = 9,\ 5y - z = 6,\ x + 5z = -3$ Set up Gauss Seidal iteration scheme for the solution. Iterate two times using initial approximations $x_0 = 1.8, y_0 = 1.2, z_0 = -0.96$	**04**
	Ans	$5x - y = 9,\ 5y - z = 6,\ x + 5z = -3$ $\therefore x = \frac{1}{5}(9 + y)$ $y = \frac{1}{5}(6 + z)$ $z = \frac{1}{5}(-3 - x)$	1
		Starting with $x_0 = 1.8, y_0 = 1.2, z_0 = -0.96$ $x_1 = 2.04$ $y_1 = 1.008$ $z_1 = -1.008$	1½
		$x_2 = 2.002$ $y_2 = 0.998$ $z_2 = -1.000$	1½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
		Important Note *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* -- --	