SUMMER – 2022 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code:** 22224

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.
8) As per the policy decision of Maharashtra State Government, teaching in English/Marathi and Bilingual (English + Marathi) medium is introduced at first year of AICTE diploma Programme from academic year 2021-2022. Hence if the students in first year (first and second semesters) write answers in Marathi or bilingual language (English +Marathi), the Examiner shall consider the same and assess the answer based on matching of concepts with model answer.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	If $f(x)=3x^2-5x+7$ show that $f(-1)=3f(1)$	**02**
	Ans	$f(-1)=15$	1
		$f(1)=5$	
		$\therefore 3f(1)=15$	
		$\therefore f(-1)=3f(1)$	1
		--	
	b)	State whether the function $f(x)=\dfrac{e^x+e^{-x}}{2}$ is odd or even	**02**
	Ans	$f(x)=\dfrac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}=\dfrac{e^{-x}+e^x}{2}$	1
		$\therefore f(-x)=f(x)$	1
		$\therefore$ function is even.	

Q. No	Sub Q. N.	Answer	Marking Scheme
1.	c)	If $y = e^{x\log_e 5}$, find $\frac{dy}{dx}$	02
	Ans	$y = e^{x\log_e 5}$	
		$\therefore y = e^{\log_e 5^x}$	½
		$\therefore y = 5^x$	½
		$\therefore \frac{dy}{dx} = 5^x \log 5$	1
		--	
	d)	Evaluate: $\int \frac{\sec^2 x}{3+\tan x}dx$	02
	Ans	Put $3+\tan x = t$	
		$\sec^2 xdx = dt$	½
		$= \int \frac{dt}{t}$	
		$= \log t + c$	1
		$= \log(3+\tan x)+c$	½
		--	
	e)	Evaluate: $\int \frac{dx}{\sqrt{9-4x^2}}$	02
	Ans	$\int \frac{dx}{\sqrt{9-4x^2}}$	
		$= \int \frac{dx}{\sqrt{4\left(\frac{9}{4}-x^2\right)}}$	
		$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\left(\frac{3}{2}\right)^2 - x^2\right)}}$ OR $= \int \frac{dx}{\sqrt{3^2-(2x)^2}}$	1
		$= \frac{1}{2}\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right)+c = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$ $= \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+c$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
1.	f)	Find the order and degree of the following differential equation:	**02**
	Ans	$\frac{d^2y}{dx^2}+\sqrt{1+\frac{dy}{dx}}=0$ $\frac{d^2y}{dx^2}+\sqrt{1+\frac{dy}{dx}}=0$ $\frac{d^2y}{dx^2}=-\sqrt{1+\frac{dy}{dx}}$ Squaring on both sides $\left(\frac{d^2y}{dx^2}\right)^2=1+\frac{dy}{dx}$ Order of D.E. $=2$, Degree of D.E. $=2$	 1 1
	g)	Find a real root of the equation $x^3-4x-9=0$ in the interval $(2,3)$ by using bisection method. $(\text{Use two iterations})$	**02**
	Ans	Let $f(x)=x^3-4x-9$ $f(2)=-9$ $f(3)=6$ $\therefore$ the root is in $(2,3)$ $x_1=\frac{a+b}{2}=\frac{2+3}{2}=2.5$ $f(2.5)=-3.375<0$ $x_2=\frac{x_1+b}{2}=\frac{2.5+3}{2}=2.75$ OR Let $f(x)=x^3-4x-9$ $f(2)=-9,\ f(3)=6\quad\therefore$ the root is in $(2,3)$	 1 1

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Q. No.	Sub Q.N.	Answer	Marking Scheme
2.			

Iteration	a(-ve)	b(+ve)	$x=\frac{a+b}{2}$	$f(x)$
I	2	3	2.5	-3.375
II	2.5	3	2.75	

Marks: 1, 1

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2 **Solve any THREE of the following :** **12**

a) If $x^4 = e^{x-4}$, then prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$ **04**

In this example y is not given so we can not find $\frac{dy}{dx}$

Given example is wrong if student replaced 4 by y give appropraite marks

Ans

If $x^y = e^{x-y}$ Show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

$x^y = e^{x-y}$

$\log x^y = \log e^{x-y}$

$y\log x = x - y\log e$ ½

$y\log x = x - y$ ½

$y\log x + y = x$

$y(\log x + 1) = x$

$y=\frac{x}{\log x+1}$

$\frac{dy}{dx}=\frac{(\log x+1).1-x.\frac{1}{x}}{(\log x+1)^2}$ 1

1

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2.		$=\dfrac{\log x}{(\log x+1)^2}$	1
		--	
	b)	If $x=2\cos t-\cos 2t,\ y=2\sin t-\sin 2t$, find $\dfrac{dy}{dx}$ at $t=\dfrac{\pi}{2}$	**04**
	Ans	$x=2\cos t-\cos 2t$	
		$\therefore \dfrac{dx}{dt}=2(-\sin t)-(-\sin 2t)2$	
		$=-2\sin t+2\sin 2t$	1
		$y=2\sin t-\sin 2t$	
		$\therefore \dfrac{dy}{dt}=2\cos t-2\cos 2t$	1
		$\therefore \dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$	
		$=\dfrac{2\cos t-2\cos 2t}{-2\sin t+2\sin 2t}=\dfrac{\cos t-\cos 2t}{-\sin t+\sin 2t}$	1
		$at\ \text{t}=\dfrac{\pi}{2}$	
		$\dfrac{dy}{dx}=\dfrac{\cos\left(\frac{\pi}{2}\right)-\cos 2\left(\frac{\pi}{2}\right)}{-\sin\left(\frac{\pi}{2}\right)+\sin 2\left(\frac{\pi}{2}\right)}$	
		$\dfrac{dy}{dx}=\dfrac{\cos\left(\frac{\pi}{2}\right)-\cos(\pi)}{-\sin\left(\frac{\pi}{2}\right)+\sin(\pi)}$	
		$\therefore \dfrac{dy}{dx}=\dfrac{(0)-(-1)}{-(1)+(0)}$	
		$=-1$	1
		--	

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2.	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length of rectangle $= x$, breadth $= y$	
		$\therefore 2x+2y=36$	
		$\therefore y=18-x$	
		Area $A = x\times y$	
		$A = x(18-x)$	
		$\therefore A = 18x - x^2$	1
		$\therefore \frac{dA}{dx} = 18-2x$	½
		$\therefore \frac{d^2A}{dx^2} = -2$	½
		Let $\frac{dA}{dx}=0$	
		$\therefore 18-2x=0$	
		$\therefore x=9$	1
		at $x=9$	
		$\frac{d^2A}{dx^2} = -2 < 0$	½
		Area is maximum at $x=9$	
		Length $x=9$; breadth y $=9$	½
	d)	Find the radius of curvature of the curve $y=\log(\sin x)$ at $x=\frac{\pi}{2}$	**04**
	Ans	$y=\log(\sin x)$	
		$\therefore \frac{dy}{dx} = \frac{1}{\sin x}.\cos x = \cot x$	1
		$\therefore \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$	1
		at $x = \pi/2$	
		$\frac{dy}{dx} = \cot\frac{\pi}{2} = 0$	½
		$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2\frac{\pi}{2} = -1$	½

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2.		$\therefore$ Radius of curvature is, $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ $\therefore \rho = \dfrac{\left[1+(0)^2\right]^{\frac{3}{2}}}{-1}$ $\therefore \rho = -1 \quad \text{i.e.} \quad 1$	1
3.		**Solve any <u>THREE</u> of the following:**	**12**
	a) Ans	Find the points on the curve $y = x^2 - 6x + 8$, where the tangent is parallel to x-axis.	**04**
		$y = x^2 - 6x + 8$ $\therefore \dfrac{dy}{dx} = 2x - 6$	1
		$\because$ Tangent is parallel to x-axis $\therefore \dfrac{dy}{dx} = 0$	1
		$\therefore 2x - 6 = 0$ $x = 3$ $\because y = x^2 - 6x + 8$ $\therefore y = 3^2 - 6(3) + 8$ $y = -1$	1
		$\therefore (3,-1)$ is the point on the curve where the tangent is parallel to x-axis.	1
	b) Ans	If $y = \tan^{-1}\sqrt{\dfrac{1+\cos x}{1-\cos x}}$, find $\dfrac{dy}{dx}$.	**04**
		$y = \tan^{-1}\sqrt{\dfrac{1+\cos x}{1-\cos x}}$	

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3.	b)	$y=\tan^{-1}\sqrt{\dfrac{2\cos^2\left(x/2\right)}{2\sin^2\left(x/2\right)}}$	1
		$y=\tan^{-1}\sqrt{\cot^2\left(x/2\right)}$	
		$y=\tan^{-1}\left(\cot\left(x/2\right)\right)$	½
		$y=\tan^{-1}\left(\tan\left(\dfrac{\pi}{2}-\dfrac{x}{2}\right)\right)$	1
		$\therefore y=\dfrac{\pi}{2}-\dfrac{x}{2}$	½
		$\therefore \dfrac{dy}{dx}=-\dfrac{1}{2}$	1
	c)	Show that the right angled triangle whose hypotenuse is 60 cm has its area maximum when each of its remaining two sides is $30\sqrt{2}$ cm.	**04**
	Ans	Let x be height of the right angled triangle	
		y be the base of right angled triangle	
		$\therefore$ By pythagoras theorem	
		$x^2+y^2=60^2$	
		$y^2=60^2-x^2$	
		$\therefore y=\sqrt{60^2-x^2}$	½
		$A=$ Area of right angled triangle $=\dfrac{1}{2}\times\text{height}\times\text{base}$	
		$\therefore A=\dfrac{1}{2}\times x\times\sqrt{60^2-x^2}$	½
		$\therefore \dfrac{dA}{dx}=\dfrac{1}{2}\left[x\dfrac{1}{2\sqrt{60^2-x^2}}(-2x)+\sqrt{60^2-x^2}\,(1)\right]$	1
		$\therefore \dfrac{dA}{dx}=\dfrac{1}{2}\left[\dfrac{-x^2}{\sqrt{60^2-x^2}}+\sqrt{60^2-x^2}\right]$	
		Put $\dfrac{dA}{dx}=0$	

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3.	c)	$\therefore \frac{1}{2}\left[\frac{-x^2}{\sqrt{60^2-x^2}}+\sqrt{60^2-x^2}\right]=0$ $\frac{-x^2}{\sqrt{60^2-x^2}}+\sqrt{60^2-x^2}=0$ $\frac{-x^2}{\sqrt{60^2-x^2}}=-\sqrt{60^2-x^2}$ $\therefore x^2=60^2-x^2$ $\therefore 2x^2=60^2$ $\therefore x=30\sqrt{2}$ $\therefore\ y=\sqrt{60^2-x^2}=\sqrt{60^2-\left(30\sqrt{2}\right)^2}=30\sqrt{2}$	1
		$\frac{d^2A}{dx^2}=\frac{1}{2}\left[-\left(\frac{\sqrt{60^2-x^2}\,(2x)-x^2\frac{1}{2\sqrt{60^2-x^2}}(-2x)}{\left(\sqrt{60^2-x^2}\right)^2}\right)+\frac{1}{2\sqrt{60^2-x^2}}(-2x)\right]$ $\frac{d^2A}{dx^2}=\frac{1}{2}\left[-\left(\frac{2x\sqrt{60^2-x^2}+\frac{x^3}{\sqrt{60^2-x^2}}}{\left(\sqrt{60^2-x^2}\right)^2}\right)-\frac{x}{\sqrt{60^2-x^2}}\right]$ $\frac{d^2A}{dx^2}=\frac{1}{2}\left[-\left(\frac{2\left(30\sqrt{2}\right)\left(30\sqrt{2}\right)+\frac{\left(30\sqrt{2}\right)^3}{\left(30\sqrt{2}\right)}}{\left(30\sqrt{2}\right)^2}\right)-\frac{\left(30\sqrt{2}\right)}{\left(30\sqrt{2}\right)}\right]$ $\therefore \frac{d^2A}{dx^2}=\frac{1}{2}\left[-(2+1)-1\right]=-2$	1
		At $x=30\sqrt{2}$ and $y=30\sqrt{2}$ $\frac{d^2A}{dx^2}<0$ its area maximum when each of its remaining two sides is $30\sqrt{2}$ cm.	

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3.	d)	Evaluate: $\int \frac{1+x-x^2}{\sqrt{x}}\,dx$	**04**
	Ans	$\int \frac{1+x-x^2}{\sqrt{x}}\,dx$	
		$=\int \frac{1+x-x^2}{x^{1/2}}\,dx$	
		$=\int \left(\frac{1}{x^{1/2}}+\frac{x}{x^{1/2}}-\frac{x^2}{x^{1/2}}\right)dx$	½
		$=\int \left(x^{-1/2}+x^{1-1/2}-x^{2-1/2}\right)dx$	1
		$=\int \left(x^{-1/2}+x^{1/2}-x^{3/2}\right)dx$	½
		$=\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+c$	1
		$=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+c$	
		$=2x^{\frac{1}{2}}+\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+c$	1
		--	
4.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Evaluate: $\int \frac{dx}{1+\sin x}$	**04**
	Ans	$=\int \frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}dx$ OR put $\tan \frac{x}{2}=t, \sin x=\frac{2t}{1+t^2}, dx=\frac{2dt}{1+t^2}$	
		$=\int \frac{1-\sin x}{1-\sin^2 x}dx$ $\quad=\int \frac{1}{1+\frac{2t}{1+t^2}}\frac{2dt}{1+t^2}$	1
		$=\int \frac{1-\sin x}{\cos^2 x}dx$ $\quad=2\int \frac{1}{1+t^2+2t}dt$	1
		$=\int \left(\frac{1}{\cos^2 x}-\frac{\sin x}{\cos^2 x}\right)dx$ $\quad=2\int \frac{1}{(1+t)^2}dt$	½

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4.	a)	$=\int \sec^2 x dx-\int \tan x \sec x dx$ $=\tan x-\sec x+c$ $=2\frac{(1+t)^{-1}}{-1}+c$ $=-2\left(1+\tan\left(x/2\right)\right)^{-1}+c$	1 ½
	b)	Evaluate: $\int \frac{(x-1)e^x}{x^2\sin^2\left(\frac{e^x}{x}\right)}dx$	**04**
	Ans	$\int \frac{(x-1)e^x}{x^2\sin^2\left(\frac{e^x}{x}\right)}dx$ Put $\frac{e^x}{x}=t$ $\therefore \frac{xe^x-e^x(1)}{x^2}dx=dt$ $\therefore \frac{(x-1)e^x}{x^2}dx=dt$ $\therefore \int \frac{1}{\sin^2 t}dt$ $=\int \operatorname{cosec}^2 t\ dt$ $=-\cot t+c$ $=-\cot\left(\frac{e^x}{x}\right)+c$	1 ½ 1 1 ½
	c)	Evaluate: $\int \tan^{-1} x dx$	**04**
	Ans	$\int \tan^{-1} x dx$ $=\int \tan^{-1}x.1dx$ $=\tan^{-1}x\int 1dx-\int\left[\frac{d\left(\tan^{-1}x\right)}{dx}.\int 1dx\right]$ $=\tan^{-1}x.x-\int \frac{1}{1+x^2}.xdx$	1 1

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4.	c)	$= x.\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$	1
		$= x.\tan^{-1}x - \frac{1}{2}\log\left(1+x^2\right)+c$	1
		--	
	d)	Evaluate: $\int_5^{10} \frac{dx}{(x-1)(x-2)}$	**04**
	Ans	$\int_5^{10} \frac{dx}{(x-1)(x-2)}$	
		Let $\frac{1}{(x-1)(x-2)} = \frac{A}{x-1}+\frac{B}{x-2}$	½
		$1 = A(x-2)+B(x-1)$	
		Put $x=1$ $\therefore A = \frac{1}{-1} = -1$	½
		Put $x=2$ $\therefore B = \frac{1}{1} = 1$	½
		$\frac{1}{(x-1)(x-2)} = \frac{-1}{x-1}+\frac{1}{x-2}$	
		$\int_5^{10} \frac{dx}{(x-1)(x-2)}$	
		$= \int_5^{10}\left[\frac{-1}{x-1}+\frac{1}{x-2}\right]dx$	
		$= \left[-\log(x-1)+\log(x-2)\right]_5^{10}$	1
		$= \left[-\log(10-1)+\log(10-2)\right]-\left[-\log(5-1)+\log(5-2)\right]$	1
		$= \left[-\log 9+\log 8\right]-\left[-\log 4+\log 3\right]$	
		$= \log\frac{8}{9} - \log\frac{3}{4}$	
		$= \log\left[\frac{8}{9}\times\frac{4}{3}\right]$	
		$= \log\frac{32}{27}$	½
		--	

Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	e)	Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt[n]{\tan x}}dx$	**04**
	Ans	$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\frac{\sqrt[n]{\sin x}}{\sqrt[n]{\cos x}}}dx$	
		Let $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x}+\sqrt[n]{\sin x}}dx ----------------(1)$	½
		$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[n]{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt[n]{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt[n]{\sin\left(\frac{\pi}{2}-x\right)}}dx$	1
		$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[n]{\sin x}}{\sqrt[n]{\sin x}+\sqrt[n]{\cos x}}dx ----------------(2)$	½
		Add (1) and (2)	
		$I+I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[n]{\cos x}}{\sqrt[n]{\cos x}+\sqrt[n]{\sin x}}dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[n]{\sin x}}{\sqrt[n]{\sin x}+\sqrt[n]{\cos x}}dx$	
		$2I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[n]{\cos x}+\sqrt[n]{\sin x}}{\sqrt[n]{\cos x}+\sqrt[n]{\sin x}}dx$	
		$2I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$	½
		$2I=\left[x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	1
		$2I=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$	
		$\therefore I=\frac{\pi}{12}$	½

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5.		**Solve any TWO of the following:**	**12**
	a)	Find the area bounded by the curve $y=\sin 2x,$ for $0\le x\le \pi$ and $x-$axis between $x=\frac{\pi}{3}$ and $x=\frac{3\pi}{4}$	**06**
	Ans	$A=\int_a^b y\,dx$	
		$A=\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} \sin 2x\,dx$	1
		$=\left[\frac{-\cos 2x}{2}\right]_{\frac{\pi}{3}}^{\frac{3\pi}{4}}$	2
		$=\frac{-1}{2}\left[\cos\left(2\times\frac{3\pi}{4}\right)-\cos\left(2\times\frac{\pi}{3}\right)\right]$	
		$=\frac{-1}{2}\left[\cos\left(\frac{3\pi}{2}\right)-\cos\left(\frac{2\pi}{3}\right)\right]$	1
		$=\frac{-1}{2}\left[0-\left(-\frac{1}{2}\right)\right]$	1
		$=\frac{-1}{4}$	
		$\therefore A=\frac{1}{4}$	1
	b)	Solve the following:	**06**
	i)	Show that $y=A\sin x+B\cos x$ is a solution of differential equation $\frac{d^2y}{dx^2}+y=0$	**03**
	Ans	$y=A\sin x+B\cos x$	
		$\therefore \frac{dy}{dx}=A(\cos x)+B(-\sin x)$	
		$\frac{dy}{dx}=A\cos x-B\sin x$	1
		$\therefore \frac{d^2y}{dx^2}=A(-\sin x)-B(\cos x)$	

Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	i)	$\frac{d^2y}{dx^2} = -[A\sin x + B\cos x]$	1
		$\therefore \frac{d^2y}{dx^2} + y = 0$	1
		--	
	ii)	Solve: $x\frac{dy}{dx} - y = x^2$	**03**
	Ans	$x \cdot \frac{dy}{dx} - y = x^2$	
		$\therefore \frac{dy}{dx} - \frac{y}{x} = x$	½
		Comparing with $\frac{dy}{dx} + Py = Q$	
		$\therefore P = -\frac{1}{x},\ Q = x$	
		$\therefore \text{I.F.} = \text{Integrating Factor} = e^{\int Pdx}$	
		$= e^{\int -\frac{1}{x}dx}$	
		$= e^{-\log_e x}$	
		$= e^{\log_e x^{-1}}$	
		$= \frac{1}{x}$	1
		$\therefore$ Solution is,	
		$y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}dx + c$	
		$\therefore \frac{y}{x} = \int x \cdot \frac{1}{x}dx + c$	
		$\therefore \frac{y}{x} = \int 1dx + c$	½
		$\therefore \frac{y}{x} = x + c$	1
		--	

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5.	c)	The acceleration of a particle is given by $\frac{d^2x}{dt^2}=3t^2-6t+8$.Find the distance covered in 2 seconds given that $v=0$ $x=0$ at $t=0$	**06**
	Ans	$\frac{d^2x}{dt^2}=3t^2-6t+8$	
		We know $\frac{d^2x}{dt^2}=\frac{dv}{dt}$	
		$\therefore \frac{dv}{dt}=3t^2-6t+8$	½
		$dv=\left(3t^2-6t+8\right)dt$	
		$\int dv=\int\left(3t^2-6t+8\right)dt$	
		$v=3\frac{t^3}{3}-6\frac{t^2}{2}+8t+c$	
		$v=t^3-3t^2+8t+c------(1)$	1
		at $t=0$,$v=0$	
		$0=0+c$	
		$c=0$	1
		From (1)	
		$v=t^3-3t^2+8t$	
		We know $v=\frac{dx}{dt}$	
		$\therefore \frac{dx}{dt}=t^3-3t^2+8t$	½
		$dx=\left(t^3-3t^2+8t\right)dt$	
		$\int dx=\int\left(t^3-3t^2+8t\right)dt$	
		$x=\frac{t^4}{4}-3\frac{t^3}{3}+8\frac{t^2}{2}+c$	1
		$\therefore x=\frac{t^4}{4}-t^3+4t^2------(2)$	
		at $t=0$,$x=0$	
		$0=0+c$	1
		$\therefore c=0$	

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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c)	From (2) $x=\frac{t^4}{4}-t^3+4t^2$ at $t=2$ $x=\frac{(2)^4}{4}-(2)^3+4(2)^2$ $\therefore x=12$ $\therefore$ Particle covered $x=12$ units in 2 seconds	1
		--	
6.		**Solve any TWO of the following:**	**12**
	a)	Solve the following:	**06**
	i)	Find the root of the equation $\cos x-xe^x=0$ using the regula-falsi method.(carry out two iterations)	**03**
	Ans	$\cos x-xe^x=0$ $\therefore f(x)=\cos x-xe^x$ $f(0)=1$ $f(1)=-2.1780$ $\therefore$ Root is in $(0,1)$	1
		By Regula -falsi method $x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{(0)(-2.1780)-(1)(1)}{-2.1780-1}=0.3147$	1
		$x_2=\frac{x_1f(b)-bf(x_1)}{f(b)-f(x_1)}=\frac{(0.3147)(-2.1780)-(1)(0.5198)}{-2.1780-0.5198}=0.4467$	1
		OR	

No. of iteration	$a(+ve)$	$b(-ve)$	$f(a)$	$f(b)$	$x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(x_1)$
1	0	1	1	-2.1780	0.3147	0.5198
2	0.3147	1	0.5198	-2.1780	0.4467	-

Marking: 2

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	ii)	Solve the following system of equations by using Gauss Elimination method. $2x+3y+z=13,\ x-y-2z=-1,\ 3x+y+4z=15.$	**03**
	Ans	$2x+3y+z=13$ and $3x+y+4z=15$ $2x-2y-4z=-2$ and $3x-3y-6z=-3$ $-$ ______ $-$ ______ $5y+5z=15$ $4y+10z=18$ $y+z=3$ $2y+5z=9$ $2y+5z=9$ $2y+2z=6$ $-$ ______ $3z=3$	
		$z=1$	1
		$y=2$	1
		$x=3$	1
		--	
	b)	Solve the following system of equations by using Gauss Seidal method $20x+y-2z=17$; $3x+20y-z=-18$; $2x-3y+20z=25.$	**06**
	Ans	$20x+y-2z=17$ $3x+20y-z=-18$ $2x-3y+20z=25$	
		$x=\frac{1}{20}(17-y+2z)$ $y=\frac{1}{20}(-18-3x+z)$ $z=\frac{1}{20}(25-2x+3y)$	1

Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	b)	Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 0.85$ $y_1 = -1.0275$ $z_1 = 1.0109$	2
		$x_2 = 1.0025$ $y_2 = -0.9998$ $z_2 = 0.9998$	2
		$x_3 = 1$ $y_3 = -1$ $z_3 = 1$	1
		--	
	c)	Using Newton-Raphson method to find the approximate root of the equation $x\log_{10} x = 1.2$. (carry out three iterations)	**06**
	Ans	$f(x) = x\log_{10} x - 1.2$ $f(2) = -0.5979 < 0$ $f(3) = 0.2314 > 0$ $\therefore$ Root is in $(2,3)$	1
		$f'(x) = x\dfrac{1}{x\log_e 10} + \log_{10} x = 0.4343 + \log_{10} x$	2
		Initial root $x_0 = 3$	
		$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 3 - \dfrac{f(3)}{f'(3)} = 2.7462$	1
		$x_2 = 2.7462 - \dfrac{f(2.7462)}{f'(2.7462)} = 2.7406$	1
		$x_3 = 2.7447 - \dfrac{f(2.7447)}{f'(2.7447)} = 2.7406$	1

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Q. No.	Sub Q. N.	Answer	Marking Scheme
6.	c)	*OR*	
		Let $f(x) = x\log_{10} x - 1.2$	1
		$f(2) = -0.5979 < 0 \qquad f(3) = 0.2314 > 0$	
		$f'(x) = 0.4343 + \log_{10} x$	1
		Initial root $x_0 = 3$	
		$x_i = x - \frac{f(x)}{f'(x)} = x - \frac{x\log_{10} x - 1.2}{0.4343 + \log_{10} x}$	
		$= \frac{x(0.4343 + \log_{10} x) - (x\log_{10} x - 1.2)}{0.4343 + \log_{10} x}$	
		$= \frac{0.4343x + x\log_{10} x - x\log_{10} x + 1.2}{0.4343 + \log_{10} x}$	
		$= \frac{0.4343x + 1.2}{0.4343 + \log_{10} x}$	1
			1
		$x_1 = 2.7462$	1
		$x_2 = 2.7406$	1
		$x_3 = 2.7406$	

		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
