17105

16172

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Figures to the right indicate full marks.

(4) Assume suitable data, if necessary.

(5) Use of Non-Programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any TEN of the following: **20**

a) Solve $\begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$

b) Find 'x' if $\begin{vmatrix} 0 & 7 & -2 \\ 11 & x & 10 \\ 4 & 8 & 1 \end{vmatrix} = 0$

c) Solve $\begin{vmatrix} 2 & 3 & x \\ 1 & 0 & 3 \\ -2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 8 \\ 2 & 1 \end{vmatrix}$

d) Define singular and non-singular matrix.

e) Define orthogonal matrix.

f) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ find $|AB|$

P.T.O.

Marks

g) Resolve into partial fractions $\frac{1}{x^2 - x}$.

h) Without using calculator, find the value of sin 75°.

i) Prove that $\frac{\sin\theta - \sin 3\theta}{\sin^2\theta - \cos^2\theta} = 2\sin\theta$

j) Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

k) Prove that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

l) Find the value of k, if the lines $kx - 6y - 9 = 0$ and $6x + 5y - 13 = 0$ are perpendicular to each other.

2. Attempt any <u>FOUR</u> of the following: **16**

a) Solve by Cramer's rule $x + y = 5, y + z = 8, z + x = 7$.

b) Solve $2\left\{\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix}\right\} = \begin{bmatrix} 260 \\ 128 \end{bmatrix}$

c) If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$, show that $A^2 - 8A$ is a scalar matrix.

d) Express the matrix $A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ as the sum of symmetric and skew-symetric materials.

e) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$

f) Resolve into partial fractions $\frac{(\tan\theta + 1)}{(\tan\theta - 1)(\tan\theta + 2)}$

Marks

3. Attempt any FOUR of the following: **16**

a) Find the adjoint of $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$.

b) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

c) Using matrix inversion method, solve

$2x + y = 3, 2y + 3z = 4, 2z + 2x = 8$

d) Resolve into partial fractions $\frac{x^2 - x + 3}{(x - 2)(x^2 + 1)}$

e) Resolve into partial fractions $\frac{2x - 3}{(x + 1)(x^2 - 1)}$

f) Revolve into partial fractions $\frac{x^4}{x^2 - 1}$

4. Attempt any FOUR of the following: **16**

a) Prove that $\sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B$

b) Prove that $\tan 40° + 2\tan 10° = \tan 50°$

c) Prove that $\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A \tan 3A$

d) Prove that $\sin 20° \sin 40° \sin 60° \sin 80° = \frac{3}{16}$

e) Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$

f) Prove that $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \cot^{-1}\left(\frac{9}{2}\right)$

Marks

5. Attempt any FOUR of the following: **16**

a) Prove that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

b) Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

c) In any triangle ABC, prove that
$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

d) Prove that $\frac{\sin 2A + 2\sin 4A + \sin 6A}{\sin A + 2\sin 3A + \sin 5A} = \cos A + \cot 3A \cdot \sin A$

e) Prove that $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$

f) Prove that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

6. Attempt any FOUR of the following: **16**

a) Find the equation of the line passing through the point of intersection of the line $2x + 3y = 13; 5x - y = 7$ and perpendicular to $3x - y + 7 = 0$

b) Find the equation of the line passing through the point of intersection of lines $x + y = 0$ and $2x - y = 9$ and through the point (2, 5).

c) If m_1 and m_2 are slopes of two lines, then prove that the acute angle between two lines is $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

d) Find the length of perpendicular from $(-3, -4)$ on the line $4(x + 2) = 3(y - 4)$.

e) Prove the perpendicular distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

f) Find the acute angle beween the lines $3x - 2y + 4 = 0$ and $2x - 3y - 7 = 0$