

22201

22223

3 Hours / 70 Marks

Seat No.

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- Instructions* –
- (1) All Questions are *Compulsory*.
 - (2) Answer each next main Question on a new page.
 - (3) Illustrate your answers with neat sketches wherever necessary.
 - (4) Figures to the right indicate full marks.
 - (5) Assume suitable data, if necessary.
 - (6) Use of Non-programmable Electronic Pocket Calculator is permissible.
 - (7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. **Solve any FIVE of the following:** **10**
- a) Define implicit function with a suitable example.
 - b) If $f(x) = x^3 - 3x + \sin x + x \cdot \cos x$ then show that $f(x) + f(-x) = 0$
 - c) Find $\frac{dy}{dx}$, if $y = (\tan^{-1}x)^2$
 - d) Evaluate $\int \frac{dx}{9 + 4x^2}$
 - e) Evaluate $\int \sin^3 x \, dx$
 - f) Find the area of the region bounded by the curve $y = 4x^2$, X axis and the ordinates $x = 0$ and $x = 2$
 - g) State Simpson's $\frac{1}{3}$ rd rule of numerical integration.

P.T.O.

2. Solve any THREE of the following:

12

- a) If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$
- b) If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$
- c) A metal wire is 60 cm long is bent to form a rectangle. Find its dimension when area is maximum.
- d) Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 1$ at $(\frac{1}{4}, \frac{1}{4})$.

3. Solve any THREE of the following:

12

- a) Find the equation of tangent and normal to the curve $4x^2 + 9y^2 = 40$ at point (1, 2)
- b) Find $\frac{dy}{dx}$, if $x^2 + y^2 = 4xy$
- c) Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} - 1}{x} \right)$
- d) Evaluate $\int \frac{(x-1)e^x}{x^2 \cdot \sin^2 \left(\frac{e^x}{x} \right)} dx$

4. Solve any THREE of the following:

12

- a) Evaluate $\int \frac{dx}{\sqrt{13 - 6x - x^2}}$
- b) Evaluate $\int \frac{dx}{5 - 4 \sin x}$
- c) Evaluate $\int \frac{x \cdot \sin^{-1} x}{\sqrt{1 - x^2}} dx$
- d) Evaluate $\int \frac{\sec^2 x}{(1 - \tan x)(2 + \tan x)} dx$
- e) Evaluate $\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+3}} dx$

5. Solve any TWO of the following:

12

- a) Find the area bounded by the parabolas $y^2 = 9x$ and $x^2 = 9y$
- b) Attempt the following
- i) Find the order and degree of the differential equation.
- $$\left(\frac{d^2y}{dx^2}\right)^{2/3} = \sqrt{y + \frac{dy}{dx}}$$
- ii) Solve the D.E. $x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$
- c) A particle starting with velocity 6 m/s has an acceleration $(1-t^2)\text{m/s}^2$. When does it first come to rest? How far has it then travelled?

6. Solve any TWO of the following:

12

- a) i) Evaluate $\int_0^4 e^x dx$, using Trapezoidal rule given that

x	0	1	2	3	4
e^x	1	2.72	7.39	20.09	54.60

- ii) Using Simpson's one third rule, Evaluate $\int_0^4 \frac{dx}{4x+5}$ taking $n = 4$ and using following table

x	0	1	2	3	4
$y = f(x)$ $\frac{1}{4x+5}$	0.2	0.1	0.076	0.058	0.047

- b) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's 1/3rd rule divide the interval $[0,1]$ into six equal parts. Find approximate value of π .
- c) Using Simpson's $\frac{3}{8}$ th rule to find $\int_0^{\pi/2} \sqrt{\cos x} dx$ with $n = 8$.
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