22210

22223

3 Hours / 70 Marks   Seat No. ☐☐☐☐☐☐☐☐

Instructions – (1) All Questions are *Compulsory*.

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. **Solve any FIVE of the following:** 10

a) If $f(x) = x^2 + 6x + 10$, find $f(2) + f(-2)$.

b) State with proof whether the function

$f(x) = \log\left(\frac{1-x}{1+x}\right)$ is even or odd.

c) Find $\frac{dy}{dx}$ if $y = e^x \cdot \tan x$.

d) Evaluate $\int e^{x \cdot \log_e a}\, dx$.

e) Evaluate $\int \sqrt{1 + \cos 2x}\ dx$.

f) Find the area bounded by the curve $y = 4x^3$, X - axis and the ordinates $x = 1$, $x = 2$.

g) Find modulus and amplitude of $1 + i\sqrt{3}$.

Marks

2. **Solve any THREE of the following:** 12

a) If $x^2 + y^2 = 4xy$, find $\frac{dy}{dx}$ at $(1, 2)$

b) Find $\frac{dy}{dx}$, if $x = a(\cos t + t \sin t)$
$$y = a(\sin t - t \cos t)$$

c) A telegraph wire hangs in the form of curve $y = a \log \left\{ \sec \left(\frac{x}{a} \right) \right\}$.
Show that curvature at any point is $\frac{1}{a} \cos \left(\frac{x}{a} \right)$.

d) A metal wire 40 cm long is bent to form a rectangle. Find it's dimensions when it's area is maximum.

3. **Solve any THREE of the following:** 12

a) Find the equations of tangents to the curve $y = x^2 - 2x - 3$ where it cuts $x -$ axis.

b) If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

c) Find $\frac{dy}{dx}$ if $y = \log \left(\frac{\sin x}{1 + \cos x} \right)$.

d) Evaluate $\int \frac{e^x (x+1)}{\cos^2 (x \cdot e^x)} dx$.

4. **Solve any THREE of the following:** 12

a) Evaluate $\int \frac{dx}{x^2 + x + 1}$.

b) Evaluate $\int \frac{dx}{5 + 3 \cos x}$.

c) Evaluate $\int \tan^{-1} x \, dx$.

d) Evaluate $\int \frac{dx}{\cos^2 x(1 - \tan x)(2 + \tan x)}$.

e) Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}} dx$.

Marks

5. **Solve any <u>TWO</u> of the following:** **12**

a) Find the area bounded by parabolas $y^2 = 9x$ and $x^2 = 9y$.

b) i) Find order and degree of the differential equation

$$\sqrt{\frac{d^2y}{dx^2}} = \sqrt[3]{\frac{dy}{dx}}$$

   ii) Solve : $\frac{dy}{dx} + y \tan x = \cos^2 x$.

c) Find the current as a function of time t using Kirchhoff's voltage law equation $L\frac{di}{dt} + Ri = E$, if resistance of 10 ohms and an inductance of 2 Henry are connected in series with a battery of 200 V.

6. **Solve any <u>TWO</u> of the following:** **12**

a) i) Express $\frac{(1+i)(2+i)}{3+i}$ in $x + iy$ form.

   ii) Find $L\{\cos 5t \cdot \cos 3t\}$.

b) Find $L^{-1}\left\{\frac{s^2 + s - 2}{s(s - 2)(s + 3)}\right\}$.

c) Solve the differential equation using Laplace transform.

$$\frac{dy}{dt} + y = t^2 \cdot e^{-t}; \ y(0) = 3.$$

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