17216

11920

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any TEN of the following: **20**

a) If $z = 1 - 3i$ Find $z^2 + 2z + 4$.

b) Find modulus and amplitude of $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

c) State whether the function $f(x) = \frac{e^x + e^{-x}}{2}$ is even or odd.

d) If $f(x) = 3x^2 - 5x + 7$ show that $f(-1) = 3f(1)$

e) Evaluate : $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

f) Evaluate : $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$

g) Evaluate : $\lim_{x \to \infty} \left[1 + \frac{2}{x}\right]^x$

P.T.O.

Marks

h) Find $\frac{dy}{dx}$ if $y = \sin(\log x) + \cos(\log x)$

i) If $x^2 + y^2 = 25$ find $\frac{dy}{dx}$

j) If $x = at^2$ and $y = 2at$ find $\frac{dy}{dx}$

k) Show that root of equation $x^2 + x - 3 = 0$ lies between 2 and 3.

l) Find the first iteration by Gauss seidal method.
$10x + y + 2z = 13, \quad 3x + 10y + z = 14, \; 2x + 3y + 10z = 15$

2. Solve any FOUR of the following : **16**

a) Express in polar form $z = -1 + \sqrt{3}\, i$

b) Simplify using De-Moiver's theorem.
$$\frac{(\cos 2\theta + i \sin 2\theta)(\cos \theta - i \sin \theta)^4}{(\cos 3\theta + i \sin 3\theta)(\cos 5\theta - i \sin 5\theta)^3}$$

c) Using Euler's formula prove that $\cosh^2\theta - \sinh^2\theta = 1$

d) Find cube root and unity by De-Moiver's theorem.

e) If $f(x) = \log(1 + \tan x)$ show that $f\left(\frac{\pi}{4} - x\right) = \log 2 - f(x)$

f) If $y = f(x) = \frac{2x - 3}{3x - 2}$ then prove that $f(y) = x$

3. Solve any FOUR of the following : **16**

a) If $f(t) = 50 \sin[100\,\pi t + 0.04]$ show that $f\left[\frac{2}{100} + t\right] = f(t)$

b) If $f(x) = \log x$ show that :

(i) $f(mn) = f(m) + f(n)$

(ii) $f\left(\frac{m}{n}\right) = f(m) - f(n)$

c) Evaluate : $\lim\limits_{x \to 0} \quad \frac{10^x - 5^x - 2^x + 1}{x^2}$

Marks

d) Evaluate : $\lim_{x \to 0} \frac{x \tan x}{1 - \cos x}$

e) Evaluate : $\lim_{x \to \infty} [\sqrt{x^2 + x + 1} - x]$

f) Evaluate : $\lim_{x \to 0} \frac{\log 10 + \log(x + 0.1)}{x}$

4. Solve any FOUR of the following : **16**

a) Using first principle of derivatives find derivatives of $f(x) = \sin x$

b) If u and v are differentiable function of x then prove that. $\frac{d}{dx} uv = u\frac{dv}{dx} + v\frac{du}{dx}$

c) If $x^3 + y^3 = 4xy$ find $\frac{dy}{dx}$.

d) Differentiate w.r.t x : $\tan^{-1}\left[\frac{5x}{1 - 6x^2}\right]$

e) If $y = (\sin^{-1} x)^x$ find $\frac{dy}{dx}$.

f) If $x = a\cos^3\theta,\ y = a\sin^3\theta$ find $\frac{dy}{dx}$

5. Solve any FOUR of the following : **16**

a) Evaluate : $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

b) Evaluate : $\lim_{x \to \infty} \left(\frac{x+1}{x-1}\right)^x$

c) Using Bisection method find the approximate root of equation $x^3 - 6x + 3 = 0$ [Three iterations only]

d) Using Regula Falsi method find the root of equation $x^3 - 9x + 1 = 0$ [Three iterations only]

e) Solve the equation $x^3 - x - 1 = 0$ using Newton Raphson method taking initial root '1' [Three iterations only]

f) Find the root of equation $x \log_e x = 1.2$ by using Bisection method. [Three iterations only]

Marks

6. Solve any FOUR of the following : **16**

a) Differentiate $\log(1+x^2)$ w.r.t. $\tan^{-1}x$

b) If $y = e^{\tan^{-1}x}$ show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$

c) Solve by Gauss - elimination method

$2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$

d) Solve by Jacobi's method

$20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$

(upto three iteration only)

e) Solve the following equations by Jacobi's method

$2x + 3y - 4z = 1$; $5x + 9y + 3z = 17$; $8x - 2y - z = 5$

(upto three iteration only)

f) With the following system of equation $5x - y = 9$, $5y - z = 6$, $x + 5z = -3$ Set up Gauss Seidal iteration scheme for the solution. Iterate two times using initial approximations, $x_0 = 1.8$, $y_0 = 1.2$, $z_0 = -0.96$.
