17105

21718

3 Hours / 100 Marks Seat No.

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any <u>TEN</u> of the following: 20

a) Find x, if $\begin{vmatrix} x & 2 \\ 8 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$

b) Find the value of the determinant

$$\begin{vmatrix} 1 & 0 & 6 \\ 7 & 2 & 5 \\ 3 & 4 & 6 \end{vmatrix}$$

c) Define: Orthogonal matrix

d) If $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$, Find 2A – 3B

Marks

e) Prove that the matrix $\begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$ is non-singular matrix.

f) Resolve into partial fraction $\dfrac{x-2}{x(x-1)}$

g) If $\sin A = \dfrac{1}{2}$, then find $\sin 3A$.

h) Prove that $\dfrac{1}{1+\sin A} + \dfrac{1}{1-\sin A} = 2\sec^2 A$

i) Express as product and evaluate $\sin 99^\circ - \sin 81^\circ$.

j) Using principal value, find the value of

$\cos^{-1}\left(-\dfrac{1}{2}\right) - \sin^{-1}\left(\dfrac{1}{2}\right)$

k) Find the slope and X-intercept of straight line,

$\dfrac{x}{4} - \dfrac{y}{3} = 2$

l) State the condition of parallel and perpendicular lines, whose slopes are m_1 and m_2.

m) Find the acute angle between the line whose slopes are $\sqrt{3}$ and $\dfrac{1}{\sqrt{3}}$.

n) Find the perpendicular distance between the point (3, 4) and the line $3x + 4y = 5$.

2. Attempt any <u>FOUR</u> of the following: 16

a) Solve by Cramer's rule

$x + y = 3$, $y + z = 5$, $z + x = 4$

b) Find x, if

$$\begin{vmatrix} x & 2 & 1 \\ 3 & x & -2 \\ 1 & 3 & 1 \end{vmatrix} = 5$$

Marks

c) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$, verify that

$(A + B)^T = A^T + B^T$

d) If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$, prove that $A^2 = 1$

e) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, verify that $(AB)' = B'A'$

f) Find x and y satisfying the matrix equation

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x & y & 3 \\ 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 7 \\ 9 & 4 & 13 \end{bmatrix}$$

3. Attempt any <u>FOUR</u> of the following: 16

a) If I is unit matrix of order 3 × 3 and

$A = \begin{vmatrix} 1 & 2 & 6 \\ 7 & 4 & 10 \\ 1 & 3 & 5 \end{vmatrix}$, Find $A^2 - 3A + I$

b) Solve the equation by inverse matrix method

$3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x + 2y + z = 4$

c) Resolve into partial fraction : $\dfrac{2x + 1}{x^2(x + 1)}$

d) Resolve into partial fraction : $\dfrac{x^2 + 1}{x^3 + 1}$

e) Resolve into partial fraction : $\dfrac{x^2}{(x + 1)(x + 2)(x + 3)}$

f) Resolve into partial fraction : $\dfrac{x^3 + x}{x^2 - 4}$

Marks

4. Attempt any FOUR of the following: 16

a) Prove that in ΔABC

$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

b) Without using calculator, prove that

$\sin 420^\circ \cos 390^\circ + \cos(-300^\circ)\sin(-330^\circ) = 1$

c) If $\tan(x+y) = \frac{3}{4}$ and $\tan(x-y) = \frac{8}{15}$ Show that

$\tan(2x) = \frac{77}{36}$

d) If A and B both obtuse angles and $\sin A = \frac{5}{13}$ and

$\cos B = \frac{-4}{5}$, then find $\sin(A+B)$.

e) Prove that $\cos A \cdot \cos(60 - A)\ \cos(60 + A) = \frac{1}{4}\cos 3A$

f) Prove that $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$

5. Attempt any FOUR of the following: 16

a) In any ΔABC, Prove that

$\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$

b) Prove that:

$$\frac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A} = \tan\left(\frac{5A}{2}\right)$$

c) Prove that :

$$\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A \tan 3A$$

d) Prove that :

$$\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{27}{11}\right)$$

Marks

e) Prove that : $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

f) Prove that : $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

6. Attempt any FOUR of the following: 16

a) Prove that : $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3) = \pi$

b) If m_1 and m_2 are the slopes of the two lines, then prove that the angle between two lines is $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1.m_2}\right|$

c) Find the equation of a straight line that passes through (3, 4) and perpendicular to the $3x + 2y + 5 = 0$.

d) Find the value of k, if the lines $kx - 6y = 9$ and $6x + 5y = 13$ are perpendicular to each other.

e) Find the equation of the line passing through the point (–2, 4) and making equal intercepts on the co-ordinate axes.

f) Find the point of intersection of lines $2x - 3y = 5$ and $6x + y = 4$.