17216

21819

3 Hours / 100 Marks Seat No. | | | | | | | | |

Instructions – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any TEN of the following: **20**

a) Find x and y if $x(1-i)+y(2+i)+6=0$.

b) If $Z = 1+i\sqrt{3}$ show that $Z^2+4=2Z$

c) If $f(x)=x^3-3x^2+5$, find $f(0)+f(2)$.

d) If $f(x)=\log(\tan x)$, find $f\left(\pi/4\right)$

e) Evaluate : $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x^2-x}\right)$

f) Evaluate : $\lim_{x\to 0}\frac{\sin 2x}{x}$

g) Evaluate : $\lim_{x\to 0}\frac{e^{3x}-1}{2x}$

P.T.O.

Marks

h) If $y = a^{2x}\cos(3x)$, find $\frac{dy}{dx}$

i) If $y = \log[\tan(4 - 3x)]$ find $\frac{dy}{dx}$

j) If $\tan^{-1}(x^2 + y^2) = a^2$ find $\frac{dy}{dx}$

k) Show that there exist a root of the equation $x^3 - 4x + 1 = 0$ in the interval (1, 2)

l) Find by Jacobi's method, the first iteration only for the following equations $5x - y = 9,\ x - 5y + z = -4,\ y - 5z = 6$

2. Solve any FOUR of the following: **16**

a) Express $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ in Polar form.

b) Simplify using De–Moiver's theorem,

$$\frac{(\cos 3\theta + i\sin 3\theta)^4\,(\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3\,(\cos 5\theta + i\sin 5\theta)^4}$$

c) Using Euler's formula, prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$

d) Prove that :

$$(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cdot\cos^n\left(\theta/2\right)\cdot\cos\left(\frac{n\theta}{2}\right)$$

e) If $f(x) = \frac{2x+5}{3x-4}$ and $t = \frac{5+4x}{3x-2}$, show that $f(t) = x$

f) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then, prove that $f\left(\frac{2x}{1+x^2}\right) = 2\,.\,f(x)$

Marks

3. Solve any FOUR of the following: **16**

a) If $f(x) = x^2 - 4x + 11$, solve the equation $f(x) = f(3x - 1)$

b) If $f(t) = 50\sin(100\pi t + 0.04)$ show that $f\left(\frac{2}{100} + t\right) = f(t)$

c) Evaluate : $\lim_{x \to 4} \frac{x^4 - 64x}{\sqrt{x^2 + 9} - 5}$

d) Evaluate : $\lim_{x \to \infty} (\sqrt{x^2 + x} - x)$

e) Evaluate : $\lim_{x \to 0} \frac{5^x + 5^{-x} - 2}{x^2}$

f) Evaluate : $\lim_{x \to 0} \left(\frac{2x + 1}{1 - 2x}\right)^{1/x}$

4. Solve any FOUR of the following: **16**

a) Using first principle find derivative of $f(x) = \log x$

b) If u and v are differentiable functions of x and $y = \frac{u}{v}$, then prove that $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

c) If $e^y = y^x$ prove that $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$

d) If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

e) If $x = 3\sin\theta - 2\sin^3\theta$, $y = 3\cos\theta - 2\cos^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

f) Differentiate $\tan^{-1}\left(\frac{2x}{1 - x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

Marks

5. Solve any FOUR of the following: **16**

a) Evaluate $\lim_{x \to 0} \frac{e^{\tan 2x} - 1}{\sin 3x}$

b) Evaluate $\lim_{x \to 0} \frac{\log(5+x) - \log(5-x)}{x}$

c) Find a real root of the equation $x^3 - 2x - 5 = 0$ using the bisection method in the interval (2, 3) carry out three iterations).

d) Using false position method, find the root of the equation $x^2 + x - 3 = 0$ in the interval (1, 2) by performing three iterations.

e) Solve $x^3 - x - 1 = 0$ by Newton Raphson method (up to three iterations.)

f) Find the root of $e^{-x} - x = 0$ by bisection method (up to three iterations.)

6. Solve any FOUR of the following: **16**

a) If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

b) Differentiate $\tan^{-1}\left(\frac{x}{1+12x^2}\right)$ w.r.t. x

c) Solve the following equations by Guass elimination method :
$x + y + z = 6, 2x - 3y + 3z = 5, 3x + 2y - z = 4$

d) Solve the following equations by Jacobi's method (take three iterations.)

$20x + y - 2z = 17, 2x - 3y + 20z = 25, 3x + 20y - z = 18.$

e) Solve the equations by Guass Seidal method (up to three iterations.)

$8x + 2y + 3z = 30, x - 9y + 2z = 1, 2x + 3y + 6z = 31$

f) With the following system of equations:

$3x + 2y = 4.5, 2x + 3y - z = 5, -y + 2z = 0.52$

g) Set up the Guass – Seidal iterations scheme for solution. Iterate two times, using initial approximations $x_0 = 0.4, y_0 = 1.6, z_0 = 0.4$

———