



MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)

(ISO/IEC -270001 – 2005 certified)

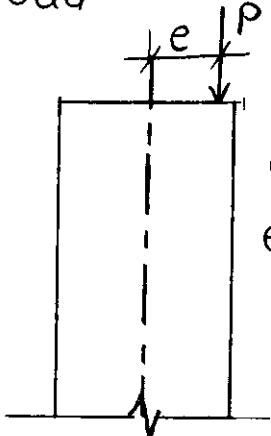
Subject code: 17422

WINTER -2016 EXAMINATION
Model Answer

Page No: 01/37

Important Instructions to examiners:

- 1) The answer should be examined by keywords and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language error such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and communication skill).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figure drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In the some cases, the assumed constants values may vary and there may be some difference in the candidates answer and model answer.
- 6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidates understanding

Question and Model Answers	Marks
<p>Q1 a)</p> <p>i) Eccentric Load → A load acts away from the centroid of the section or a load whose line of action do not coincide with the axis of member is called as the eccentric load</p>  <p>P = Applied load e = Eccentricity</p>	<p>01 mark</p> <p>01 mark</p>

ii) The differential equation for slope-

$$\frac{dy}{dx} = \int \frac{Mx}{EI}$$

$$y = \int \frac{dy}{dx} \cdot \frac{1}{EI}$$

$$\frac{dy}{dx} = \text{Slope}$$

$Mx = \text{loading}$

$y = \text{deflection}$

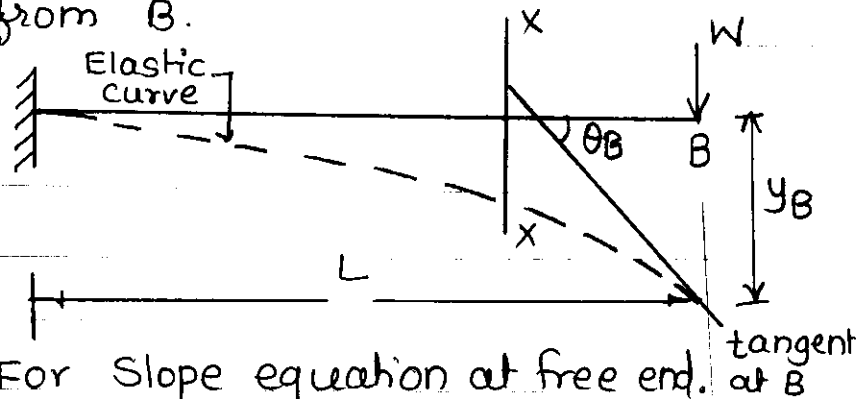
The differential equation for deflection-

$$y = \iint \frac{Mx}{EI}$$

of mark

of mark

iii) Consider a section x-x at a distance x from B.



• For Slope equation at free end. tangent at B

$$\frac{dy}{dx_B} = -\frac{WL^2}{2EI} \text{ ----- Slope equation}$$

of mark

• For Deflection equation at free end

$$y_B = y_{max} = -\frac{WL^3}{3EI}$$

of mark

iv) Macaulay's method is used for finding slope and deflection of beam as follows.

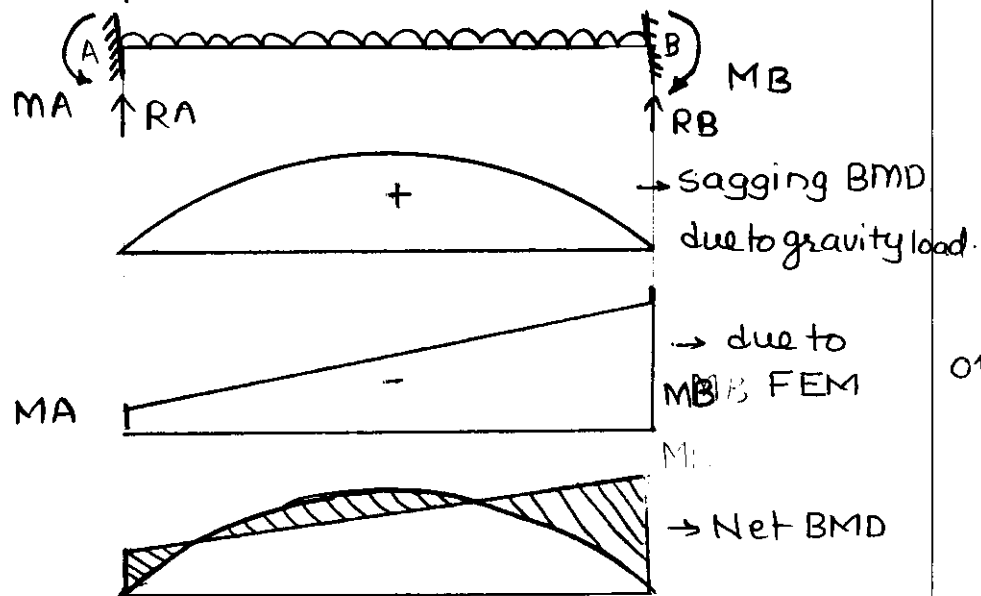
a. Use of Macaulay's method is very convenient for cases of discontinuous and or discrete loading.

01 mark

b. Typically udl & vdl over the span and no. of concentrated loads are conveniently handle using this technique.

01 mark

v) Principle of Superposition →



01 m

The sagging diagram due to gravity load neutralized the hogging diagram due to fixed end moments to some

01 m

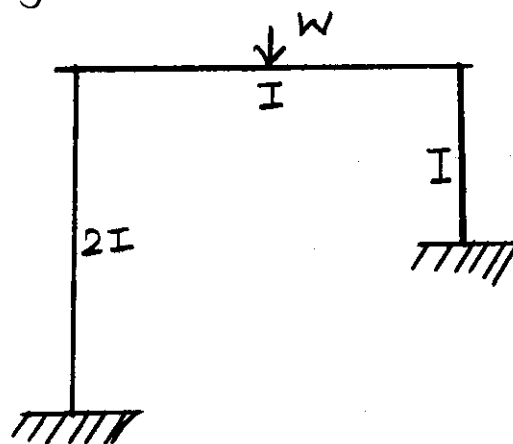
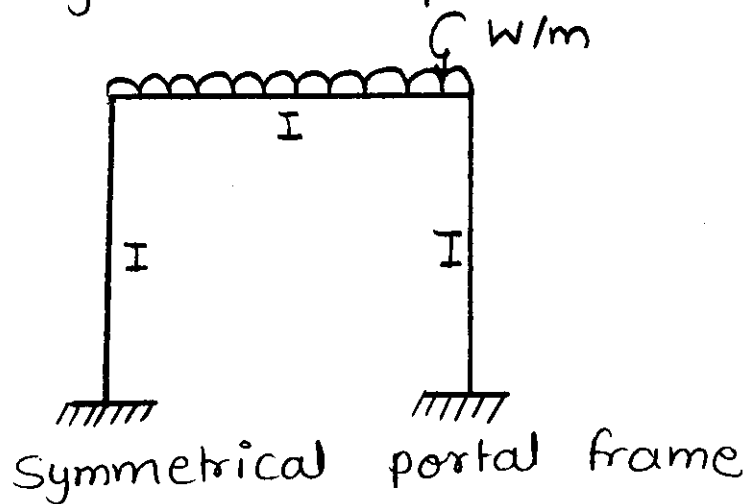
Continued.

The remaining diagram indicates the nature and magnitude of net bending moment in the beams, Hence it is named as principle of superposition

vii) Different types of portal frames can be classified as

a) Symmetrical portal frames

b) Unsymmetrical portal frames.

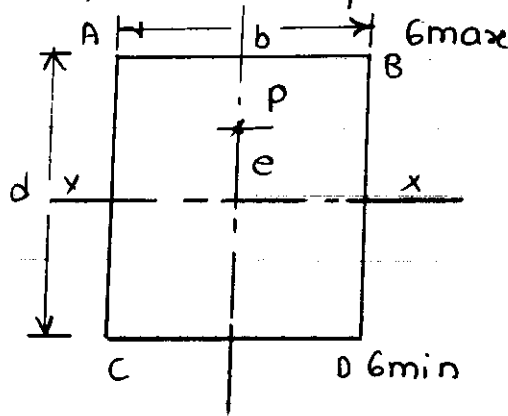


Unsymmetrical portal frame

	<p>vii) stiffness factor: → It is the moment required to produce unit rotation at an end, without translating it.</p>	01m
	<p>Distribution factor: → It is the ratio of relative stiffness of a members to the total stiffness of all the members meeting at a joint.</p>	01m
	<p>viii) Perfect frame: → A frame made up of just sufficient no. of members so that it can remain in stable equilibrium, when loaded at joints. $n = 2j - 3$ → perfect.</p>	01m
	<p>Imperfect frame: → A frame made up of either more than or less than, just sufficient number of members, to keep it in static equilibrium is called imperfect frame. $n < 2j - 3$ Deficient $n > 2j - 3$ Redundant</p>	01m

Q1. b)

i) Limit of eccentricity for rectangular section, $d = \text{depth}$



$$\sigma_{\max} = \sigma_0 + \sigma_b$$

$$\sigma_{\min} = \sigma_0 - \sigma_b$$

$$= \frac{P}{A} - \frac{M}{I_x} \times y$$

Where $M = P \times e$

$$I_x = \frac{bd^3}{12}$$

$$y = d/2$$

$$\therefore \sigma_{\min} = \frac{P}{A} - \frac{Pe}{(bd^3/12)} \times \frac{d}{2}$$

\therefore For No tension $\sigma_{\min} = 0$

$$0 = \frac{P}{A} - \frac{Pe}{bd^2} \times 6$$

$$\frac{Pe}{bd^2} \times 6 = \frac{P}{A}$$

\therefore $e \leq d/6$ Limit of eccentricity for rectangular sec.

01 m

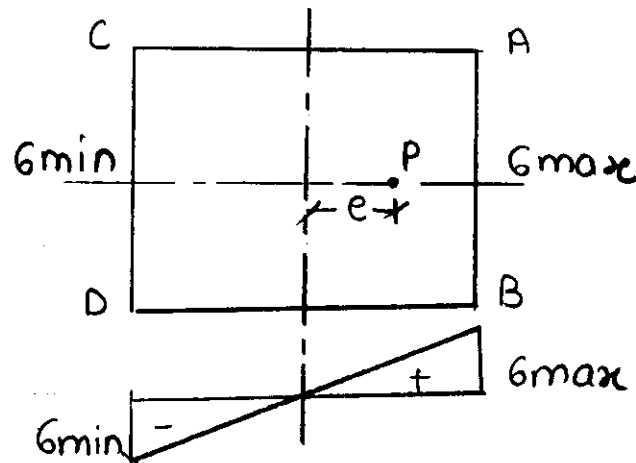
01 m

01 m

01 m

ii)

Minimum and maximum stresses developed at the base of the section



i) $\sigma_d = \text{direct stress} = \frac{P}{A}$

ii) $\sigma_b = \frac{M}{I} \times y$

On face AB $\sigma_{max} = \sigma_d + \sigma_b = +ve$

CD $\sigma_{min} = \sigma_d - \sigma_b = -ve$

For Eccentric loading.

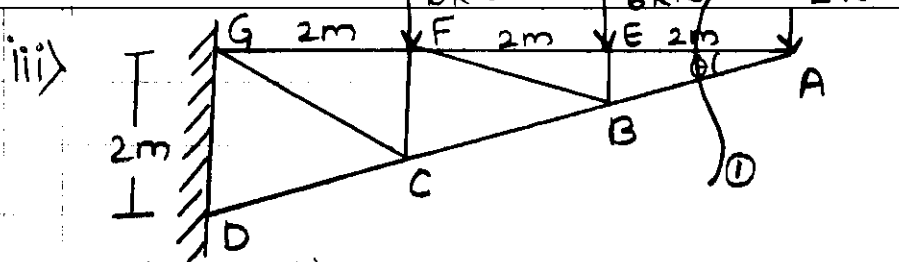
$$\sigma_{max} = \frac{P}{A} + \frac{Pe}{I} \times y$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$\therefore \sigma_{max} = \text{direct stress} + \text{Bending stress}$

$$\sigma_{max} = \frac{P}{A} + \frac{Pe}{I} \times y$$

$$\sigma_{min} = \frac{P}{A} - \frac{Pe}{I} \times y$$



Take section (1)-(1) such that it passes through members whose force are required

$$\tan \theta = \frac{2m}{6m}$$

$$\theta = 18.43^\circ$$

For equilibrium $\sum M_B = 0$

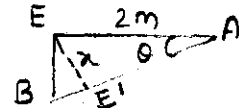
$$2 \times 2 - F_{AE} \times 0.66$$

$$F_{AE} = 6.0 \text{ kN Tensile}$$

$$\sum M_E = 0$$

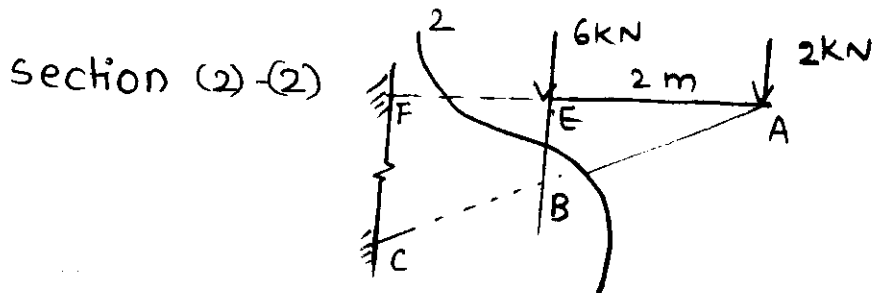
$$2 \times 2 + F_{AB} \times 0.632$$

$$F_{AB} = -6.32 \text{ kN (C)}$$



$$\sin \theta = \frac{EE'}{2}$$

$$EE' = 0.632 \quad 01$$



Assuming EF & BE tensile

$$\sum M_A = 0$$

$$-6 \times 2 - BE \times 2 = 0$$

$$\therefore BE = -6 \text{ kN (Comp)}$$

$$\sum M_B = 0$$

$$-FE \times 0.66 + 2 \times 2$$

$$FE = 6.00 \text{ kN (Tensile)}$$

$$\sin \theta = \frac{h}{4}$$

$$h = \frac{2}{1.26}$$

01

Section (3)-(3)

Assuming FB & BC are tensile

$$\sum M_A = 0$$

$$h = 1.26$$

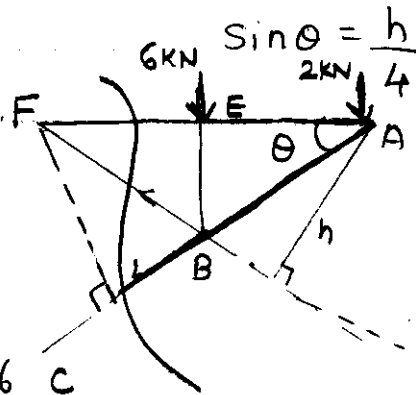
$$FB \times 1.26 - 6 \times 2$$

$$FB = 9.52 \text{ kN (T)}$$

$$\sum M_F = 0$$

$$6 \times 2 + 2 \times 4 + BC \times 1.26 = 0$$

$$BC = -15.87 \text{ kN (C)}$$



01

01

Sl No.	Member	Force	Nature
1	FE	6.00 kN	Tensile
2	FB	9.52 kN	Tensile
3	CB	-15.87 kN	Compressive

		mark
Q2. a)	Givendata.	
	dia = 250 mm	
	P = 200 kN	
	e = 150 mm	
	W = ?	
	$6_{min} = 6d - 6b$	01
	For No tension, $6_{min} = 0$	01
	$\therefore 0 = 6d - 6b$	
	$\therefore 6d = 6b$	
	$\frac{P}{A} = \frac{P \cdot e}{I} \times y$	01
	Where,	
	$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 250^2 = 49087.38$	
	$I = \frac{\pi}{64} \times 250^4 = 191.747 \times 10^6 \text{ mm}^4$	
	$\therefore \frac{(W+200)}{A} = \frac{200 \times 10^3 \times 150}{191.747 \times 10^6} \times \frac{250}{2}$	
	$W = 760 \text{ kN}$	01
	\therefore To avoid the tensile stress at the base, $W = 760 \text{ kN}$	

b) Data: $\rightarrow P = 250 \text{ kN}$

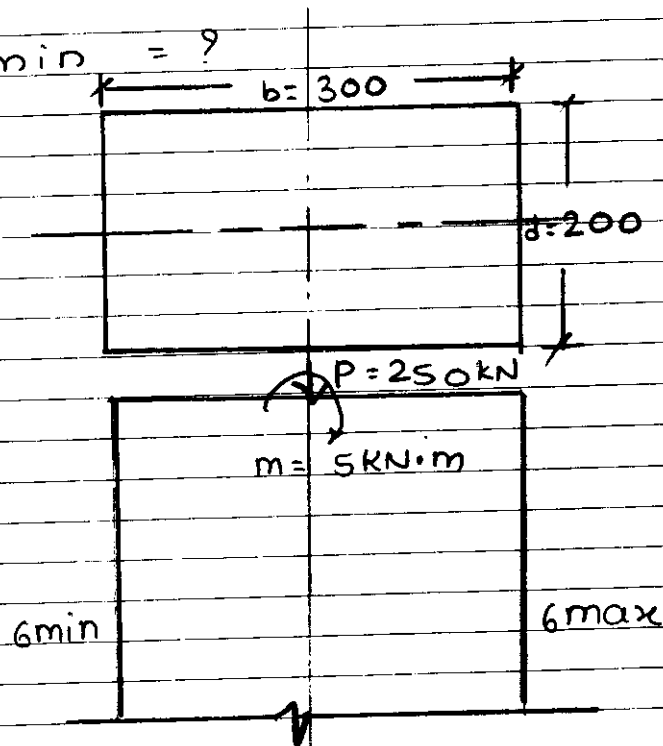
$$b = 300 \text{ mm}$$

$$d = 200 \text{ mm}$$

$$m = 5 \text{ kN}\cdot\text{m}$$

$$\sigma_{\text{max}} = ?$$

$$\sigma_{\text{min}} = ?$$



Where,

$$A = 300 \times 200 = 60000 \text{ mm}^2$$

$$I_y = \frac{200 \times 300^3}{12} = 450 \times 10^6 \text{ mm}^2$$

$$\sigma_{\text{max}} = \sigma_d + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{I} \times y$$

$$= \frac{250 \times 10^3}{60000} + \frac{5 \times 10^6}{450 \times 10^6} \times \frac{300}{2}$$

$$= 5.82 \text{ N/mm}^2 \text{ (Compressive)}$$

$$\sigma_{\min} = \sigma_d - \sigma_b$$

$$= \frac{P}{A} - \frac{M}{I} \times y$$

$$= 4.16 - 1.66$$

$$= 2.49 \text{ N/mm}^2 \text{ (Compressive)}$$

01

a) Data,

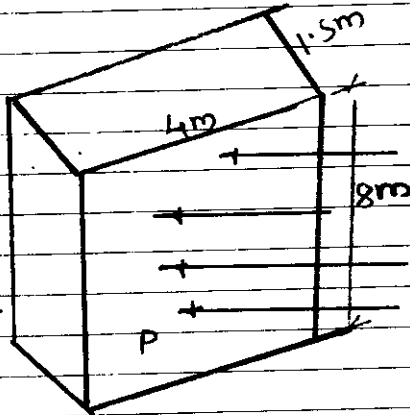
$$h = 8 \text{ m}$$

$$b = 4 \text{ m}$$

$$t = 1.5 \text{ m}$$

$$p_w = 2.5 \text{ kN/m}^2$$

$$s = 24 \text{ kN/m}^3$$



Wind Pressure acting on 4m side,

→ To find maximum & minimum stress,

1) self wt of wall $W = s \times \text{Volume}$

$$= 24 [4 \times 1.5 \times 8]$$

$$= 1152 \text{ kN}$$

01

2) Wind Force $P = c_p \cdot \text{Area of wall}$

$$= 2.5 \times [4 \times 8]$$

$$= 80 \text{ kN}$$

3) Area at base $= 4 \times 1.5 = 6 \text{ m}^2$

01

4) $I_y = \frac{4 \times 1.5^3}{12} = 1.125 \text{ m}^4$

5) stresses,

$$\sigma_{\max} = \sigma_d + \sigma_b$$

$$\begin{aligned} \sigma_{\max} &= \frac{W}{A} + \frac{P \cdot H/2}{I} \times y \\ &= \frac{1152}{6} + \frac{80 \times 8/2}{1.125} \times \frac{1.5}{2} \\ &= 192 + 213.33 \end{aligned}$$

$$\sigma_{\max} = 405.33 \text{ N/mm}^2 \text{ (Comp)}$$

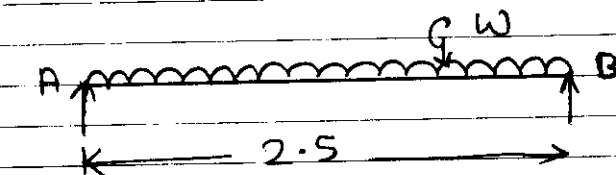
01

$$\begin{aligned} \sigma_{\min} &= \frac{W}{A} - \frac{P \cdot H/2}{I} \times y \\ &= 192 - 213.33 \end{aligned}$$

$$\sigma_{\min} = 21.33 \text{ N/mm}^2 \text{ (Tensile)}$$

01

d)



Data: →

$$\text{slope} = 1.5^\circ$$

maximum deflection = $y = ?$

where,

$$\frac{dy}{dx} = \text{slope} = 1.5^\circ$$

$$1.5^\circ = \frac{wL^3}{24EI}$$

$$1.5 \times \frac{\pi}{180} = \frac{w \times 2.5^3}{24EI}$$

$$w = 0.040EI$$

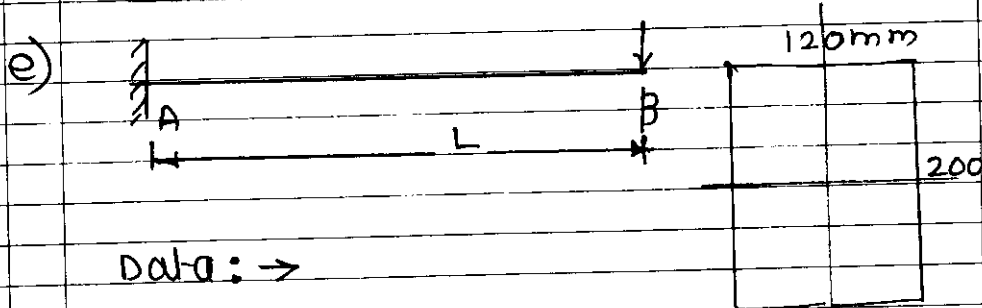
02

$$\text{but } y_{\max} = \frac{-5wL^4}{384EI}$$

$$y_{\max} = \frac{-5 \times 0.040 EI \times 2.5^4}{384 EI}$$

$$= 0.02045 \text{ m}$$

$$\therefore y_{\max} = 20.45 \text{ mm downward} \quad 02$$



Data: →

$$W = 6 \text{ kN}$$

$$\frac{dy}{dx} = \text{Slope} = 1.5 \times 10^{-3} \text{ radians}$$

$$E = 100 \text{ kN/mm}^2$$

$$= 1 \times 10^8 \text{ kN/m}^2 \quad 01$$

Where,

$$I = \frac{120 \times 200^3}{12} = 80 \times 10^6 \text{ mm}^4 \quad 01$$

$$= 80 \times 10^{-6} \text{ m}^4$$

$$\therefore \text{Slope} = \frac{dy}{dx} = \frac{WL^2}{2EI} \quad 01$$

$$1.5 \times 10^{-3} = \frac{6 \times L^2}{2 \times 1 \times 10^8 \times 80 \times 10^{-6}}$$

$$L^2 = 4$$

$$\therefore L = 2 \text{ m} \quad 01$$

F) Clapeyron's Theorem :->

Let ABC be a continuous beam, selected for analysis and loaded as shown below.

a_1 & a_2 - are the area of BMD

$$M_A = M_C = 0$$

∴ The supports are simple support

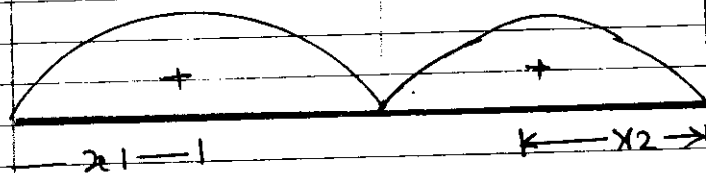
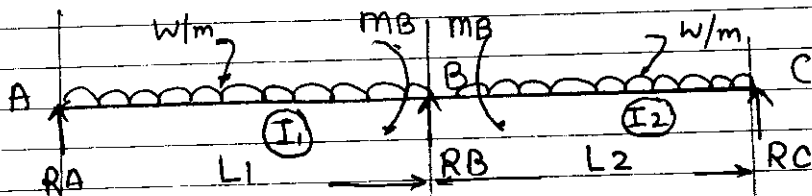
To obtain MB Clapeyron's thm.

01 m

$$\frac{M_A L_1}{I_1} + 2M_B \left[\frac{L_1}{I_1} + \frac{L_2}{I_2} \right] + \frac{M_C L_2}{I_2}$$

$$= -6 \left[\frac{a_1 x_1}{L_1 I_1} + \frac{a_2 x_2}{L_2 I_2} \right]$$

01 m



01 m

Sagging BMD

Where,

$$M_A \text{ \& } M_C = 0$$

I_1 & I_2 are the moment of Inertia of beam

M_B can be obtained from the above equation.

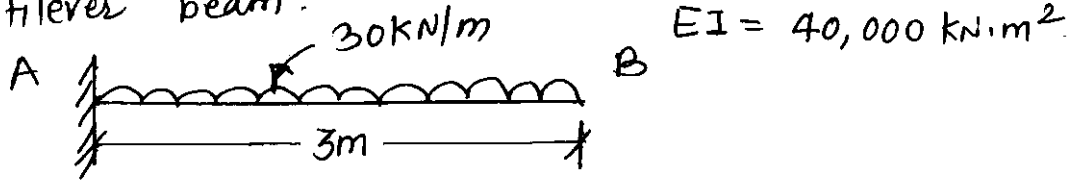
L_1 & L_2 are span

x_1 & x_2 are distance of c.g of BMD

01 m

Q. 3 Attempt any FOUR of the following.

3(a) Calculate Deflection and slope at free end for cantilever beam.



1) Deflection at free end

$$y_B = y_{max} = -\frac{WL^4}{8EI}$$

$$= -\frac{30 \times (3)^4}{8 \times 40,000}$$

1 (Mark)

$$[y_{max} = -7.593 \text{ mm}] \text{ -ve sign indicate Downward deflection}$$

1 Mark

2) Slope at free end -

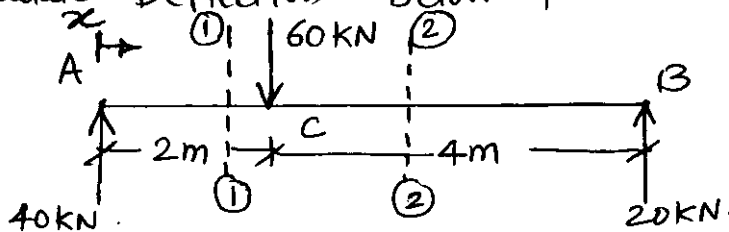
$$\theta_B = \frac{WL^3}{6EI}$$

$$\theta_B = \frac{30 \times 3^3}{6 \times 40,000} = 0.003375 \text{ radians}$$

1 Mark

1 Mark

3(b) Calculate Deflection below point load. Using Macaulay's Method.



1) To Calculate Support Reaction.

$$\sum M_A = 0 \quad (\curvearrowright +)$$

$$R_A + R_B = 60 \text{ kN}$$

$$-R_B \times 6 + 60 \times 2 = 0$$

$$R_B = \frac{-120}{-6}$$

$$R_B = 20 \text{ kN}$$

$$\therefore R_A = 60 - 20 = 40 \text{ kN}$$

2) $EI \frac{d^2y}{dx^2} = M$ - Differential equation

$$EI \frac{d^2y}{dx^2} = 40x \quad \left| \quad -60(x-2) \right|$$

$x = 2 \text{ m} \quad \quad \quad x = 6 \text{ m}$

Question & sub Question

Answer

Mark

Integrating With respect to x

$$EI \frac{dy}{dx} = \frac{40x^2}{2} + C_1 \Big|_{x=2m} - \frac{60(x-2)^2}{2} \Big|_{x=6m} \quad \text{--- slope eq}^n$$

1 Mark

Again Integrating w.r. to x .

$$EI \cdot y = \frac{20x^3}{3} + C_1 x + C_2 \Big|_{x=2m} - \frac{60(x-2)^3}{6} \quad \text{--- Deflection eq}^n$$

1 Mark

3) Calculation of Constants of Integration
Boundary Condition.

$x=0, y=0$ putting in Deflection equation

$$0 = 0 + C_2 - 10(0-3)^3 \quad \text{Neglecting -ve terms.}$$

$$\boxed{C_2 = 0}$$

At $x=6m, y=0$ putting in deflection equation

$$EI(0) = \frac{20}{3}(6)^3 + 6C_1 + 0 - 10(6-2)^3$$

$$0 = 1440 + 6C_1 - 640$$

$$\boxed{C_1 = -133.333}$$

1 Mark

4) To Calculate Deflection below point load

At $x=2m, y=y_c$ putting in Deflection eqⁿ with Constants of Integration.

$$EI \cdot y_c = \frac{20}{3}(2)^3 - 133.333(2) + 0 - 10(2-2)^3$$

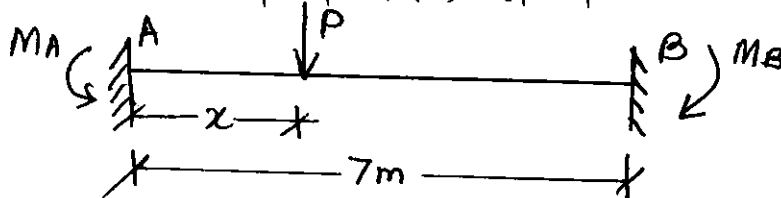
$$y_c = \frac{53.34 - 266.66}{EI}$$

$$\boxed{y_c = -\frac{213.32}{EI}}$$

1 Mark

Q3(c)

Calculation of position of point load for fixed Beam



$$M_A = 2M_B$$

1 Mark

Question
& Sub
Question

Answer

Marks

Fixed End moments.

$$M_A = - \frac{Wab^2}{L^2} = \frac{-P \times x \times (7-x)^2}{(7)^2}$$

1 Mark

$$M_B = - \frac{Wa^2b}{L^2} = \frac{-P \times x^2 \times (7-x)}{(7)^2}$$

1 Mark

Putting values of M_A & M_B in eqⁿ (I)

$$\frac{-P \times x \times (7-x)^2}{7^2} = 2 \left[\frac{-P \times x^2 \times (7-x)}{7^2} \right]$$

$$(7-x) = 2x$$

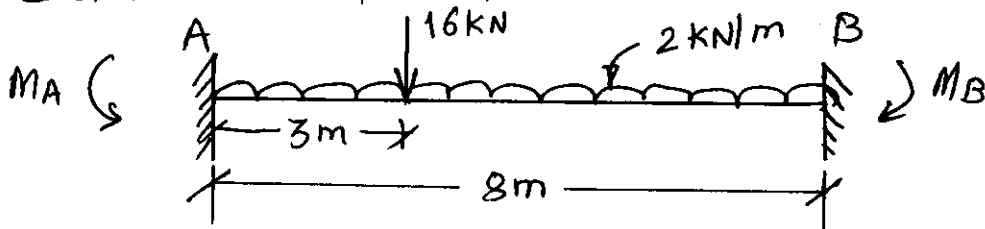
$$3x = 7$$

$$[x = 2.333\text{m}]$$

1 Mark

Q.3(d)

Calculation of fixed end moments.



Fixed End moments

$$M_A = M_{A1} + M_{A2} = - \frac{WL^2}{12} - \frac{Wab^2}{L^2}$$

1 Mark

$$M_A = \frac{-2 \times 8^2}{12} - \frac{16 \times 3 \times 5^2}{8^2}$$

$$M_A = -10.67 - 18.75$$

$$[M_A = -29.42 \text{ kN}\cdot\text{m}]$$

1 Mark

$$M_B = M_{B1} + M_{B2} = - \frac{WL^2}{12} - \frac{Wa^2b}{L^2}$$

$$M_B = \frac{-2 \times 8^2}{12} - \frac{16 \times 3^2 \times 5}{8^2} = -10.67 - 11.25$$

$$[M_B = -21.92 \text{ kN}\cdot\text{m}]$$

1 Mark

Question
L. Sub.
Question

Answer

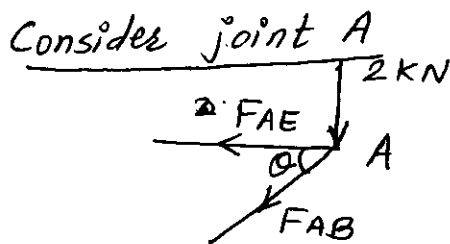
Marks

Q.3(e) Assumptions made in Analysis of simple frames

- i) The members are pin jointed at ends.
- ii) The frame is perfect frame i.e. it satisfies $n = 2j - 3$ Condition.
- iii) The loads are acting at joints only.
- iv) Self wt. of members is neglected.
- v) Only Axial forces (tensile and compressive) are induced in the member.

Any 4
1x4
= 4 marks.

Q.3f. Find forces in member AB, AE, EB & EF using method of joint.



Consider ΔADG in fig. 1.

$$\tan \theta_A = \frac{2}{6}$$

$$\theta_A = \tan^{-1}(0.333)$$

$$[\theta_A = 18.433^\circ]$$

Considering equations of equilibrium

$$\sum F_x = 0$$

$$-F_{AE} + F_{AB} \cos \theta = 0$$

$$\theta = 18.433^\circ$$

$$\sum F_y = 0 \quad -F_{AE} - 0.948 F_{AB} = 0 \quad \text{--- (I)}$$

$$-2 - F_{AB} \sin(18.433) = 0$$

$$[F_{AB} = -6.325 \text{ kN}] \quad \text{-ve sign indicates Compression}$$

1 Mark

Putting F_{AB} in eqⁿ (I)

$$-F_{AE} - 0.948(-6.325) = 0$$

$$[F_{AE} = +6 \text{ kN}] \quad \text{+ve sign indicates Tension.}$$

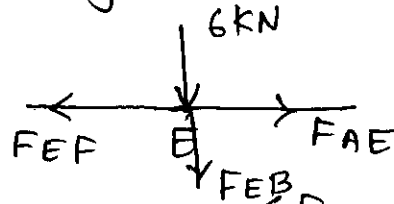
1 Mark

Question
& Sub
Question

Answer

Marks

Consider joint E



For equilibrium $\sum F_x = 0$

$$-F_{EF} + 6 = 0$$

$$\therefore [+F_{EF} = +6 \text{ kN}] \text{ +ve sign indicate Tension}$$

1 Mark

$$\sum F_y = 0$$

$$-6 - F_{EB} = 0$$

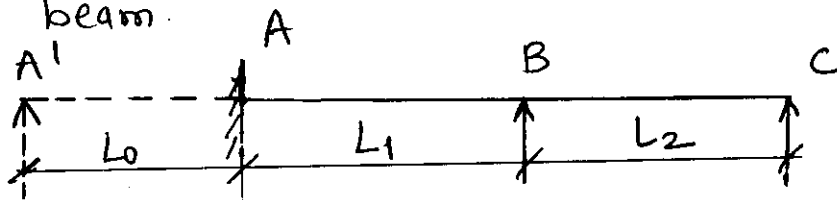
$$[F_{EB} = -6 \text{ kN}] \text{ -ve sign indicate Compression}$$

1 Mark

Q. 4 Attempt Any FOUR of the following.

4(a) Concept of imaginary zero span :-

This concept is used when one end of continuous beam is fixed and solve Problem using clapeyron's Theorem. Consider following Continuous beam.



2 mark

When the one of the end of continuous beam is fixed as shown in fig. an imaginably span A'A is considered whose length is zero and the load acting on span is also zero such type of span is called zero span.

1 mark

First clapeyron's theorem is applied to span A'A and AB and then to span AB & BC to get two equations for M_A & M_B . These two equations can be solved simultaneously to obtain support moments M_A & M_B

1 mark

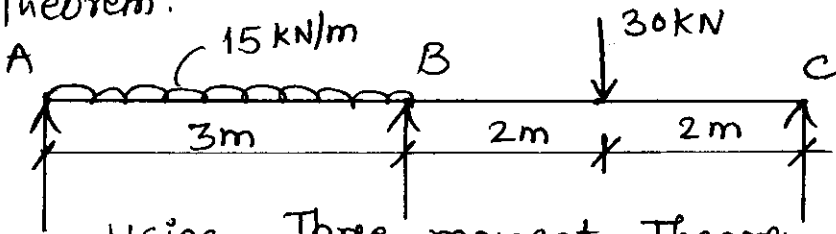
Question & sub Question

Answer

Marks

4(b)

Calculate support moment using Three moment Theorem.



Using Three moment Theorem.

$$M_A \cdot L_1 + 2M_B(L_1 + L_2) + M_C(L_2) = - \left[\frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} \right] \quad 1 \text{ mark}$$

To calculate S.S. B.M.

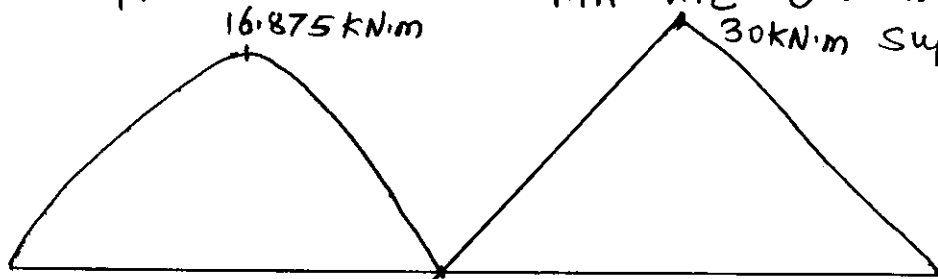
Consider span AB

$$M_{\max} = \frac{wL^2}{8} = \frac{15 \times 3^2}{8} = 16.875 \text{ kN.m}$$

Span BC

$$m_{\max} = \frac{wL}{4} = \frac{30 \times 4}{4} = 30 \text{ kN.m} \quad 1 \text{ mark}$$

Support moments $M_A = M_C = 0$ - Simple supports.



A_1 = Area of B.M. diag for span AB

$$A_1 = \frac{2}{3} \times b \times h = \frac{2}{3} \times 16.875 \times 3 = 33.75 \text{ kN.m}^2$$

A_2 = Area of B.M. diag for span BC

$$A_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4 \times 30 = 60 \text{ kN.m}^2$$

$$\bar{x}_1 = \frac{3}{2} = 1.5 \text{ m} \quad \bar{x}_2 = \frac{4}{2} = 2 \text{ m} \quad 1 \text{ mark}$$

$$0 + 2M_B(3+4) + 0 = - \left[\frac{6 \times 33.75 \times 1.5}{3} + \frac{6 \times 60 \times 2}{4} \right]$$

$$2M_B(7) = - [101.25 + 180]$$

$$M_B = -20.089 \text{ kN.m} \quad 1 \text{ mark}$$

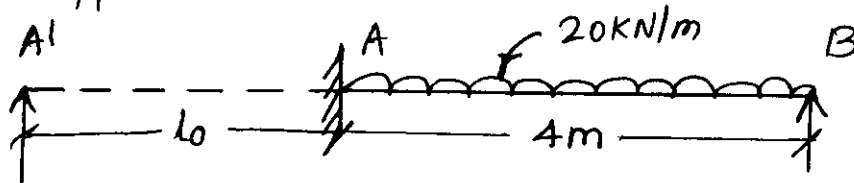
Question
 & sub
 Question

Answer

Marks

Q.4(c)

Propped Cantilever Beam.



Consider the imaginary span AA' at fixed support having zero span and loading. Apply Clapeyron's Theorem for span A'A and AB

$$M_{A'}(L_0) + 2M_A(L_0 + L_1) + M_B(L_1) = - \left[\frac{6A_0\bar{X}_0}{L_0} + \frac{6A_1\bar{X}_1}{L_1} \right] \quad -1 \text{ mark}$$

Simply Supported BMD.

for span A'A = 0

Span AB $M_{max} = \frac{20 \times 4^2}{8} = 40 \text{ kN}\cdot\text{m}$.

$A_0 = 0, \bar{X}_0 = 0, A_1 = \frac{2}{3}bh = \frac{2}{3} \times 40 \times 4$

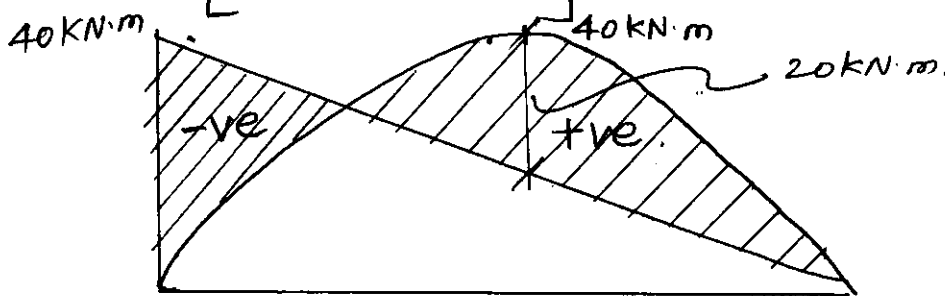
$A_1 = 106.67 \text{ kN}\cdot\text{m}^2$

$\bar{X}_1 = \frac{4}{2} = 2 \text{ m}$.

$M_B = 0$ - For simple support

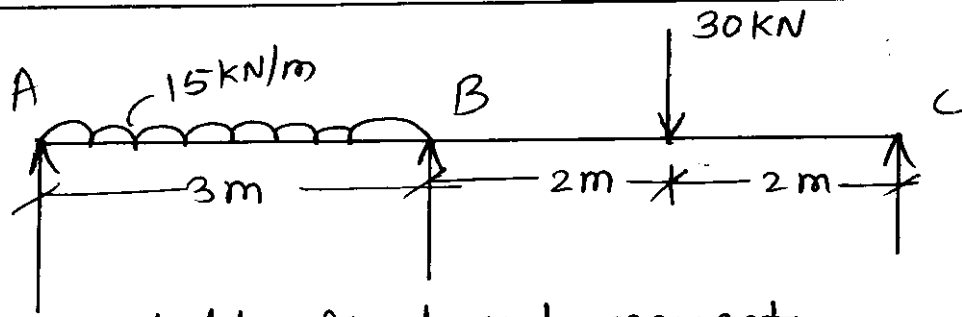
$$2M_A(0 + 4) = - \left[\frac{6 \times 0 \times 0}{0} + \frac{6 \times 106.67 \times 2}{4} \right]$$

$[M_A = -40 \text{ kN}\cdot\text{m}]$



FINAL BMD.

Q: 4(d)



1) To calculate fixed end moments.

Span AB

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{15 \times 3^2}{12} = -11.25 \text{ kN}\cdot\text{m}$$

$$M_{FBA} = +\frac{WL^2}{12} = +11.25 \text{ kN}\cdot\text{m}$$

Span BC

$$M_{FBC} = -\frac{WL}{8} = -\frac{30 \times 4}{8} = -15 \text{ kN}\cdot\text{m}$$

$$M_{FCB} = +\frac{WL}{8} = \frac{30 \times 4}{8} = +15 \text{ kN}\cdot\text{m}$$

2) Calculate Distribution Factor

Joint	Member	Stiffness Factor	Total Stiffness	Distribution factor
B	BA	$K_{BA} = \frac{3EI}{L}$ $K_{BA} = \frac{3}{3} = 1EI$	T.S. = 1 + 0.75 = 1.75EI 1.75EI	D.F. = $\frac{1}{1.75}$ D.F. = 0.571
	BC	$K_{BC} = \frac{3EI}{L}$ $K_{BC} = \frac{3EI}{4} = 0.75EI$		

	A	B	C	Remark
		0.571	0.429	
	-11.25	+11.25	-15	+15
	+11.25	+5.625	-7.5	-15
	0	+16.875	-22.5	0
		+3.211	+2.413	
	0	+20.086	-20.086	0

$M_B = 20.086 \text{ kN}\cdot\text{m}$

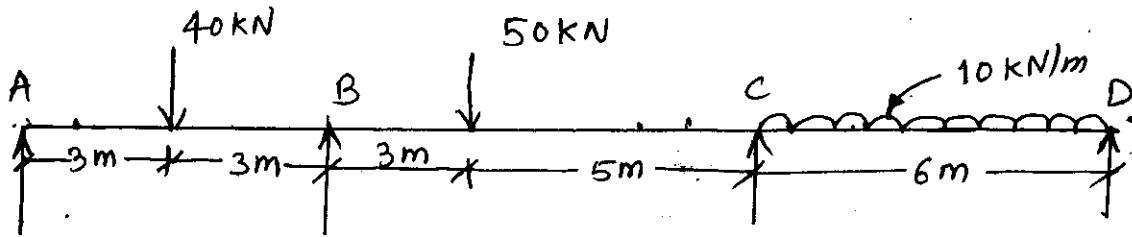
Hogging moment

1 mark

1 mark

2 marks

Q.4(e)



1) To calculate fixed end moments.

$$M_{FAB} = -\frac{WL}{8} = -\frac{40 \times 6}{8} = -30 \text{ kN}\cdot\text{m}$$

$$M_{FBA} = +\frac{WL}{8} = +30 \text{ kN}\cdot\text{m}$$

$$M_{FBC} = -\frac{Wab^2}{L^2} = -\frac{50 \times 3 \times 5^2}{8^2} = -58.59 \text{ kN}\cdot\text{m}$$

$$M_{FCB} = +\frac{Wa^2b}{L^2} = \frac{50 \times 3^2 \times 5}{8^2} = 35.156 \text{ kN}\cdot\text{m}$$

$$M_{FCD} = -\frac{10 \times 6^2}{12} = -30 \text{ kN}\cdot\text{m}$$

$$M_{FDC} = \frac{10 \times 6^2}{12} = +30 \text{ kN}\cdot\text{m}$$

1 Mark

2) To calculate Distribution factors

Joint	Member	Stiffness factor	Total stiffness	Distribution Factor (D.F.)
B	BA	$k_{BA} = \frac{3EI}{L} = \frac{3EI}{6}$ $k_{BA} = 0.5EI$	1EI	$DF_{AB} = \frac{0.5EI}{1EI}$ $DF_{AB} = 0.5$
	BC	$k_{BC} = \frac{4EI}{L} = \frac{4EI}{8}$ $k_{BC} = 0.5EI$		$DF_{BC} = \frac{0.5EI}{1EI}$ $DF_{BC} = 0.5$
C	CB	$k_{CB} = \frac{4EI}{L} = \frac{4EI}{8}$ $k_{CB} = 0.5EI$	1EI	$DF_{CB} = \frac{0.5EI}{1EI}$ $DF_{CB} = 0.5$
	CD	$k_{CD} = \frac{3EI}{6}$ $k_{CD} = 0.5EI$		$DF_{CD} = \frac{0.5}{1}$ $DF_{CD} = 0.5$

1 Mark

Question & Sub Question	Answer	Marks
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3) Calculation of Moment Distribution Method.

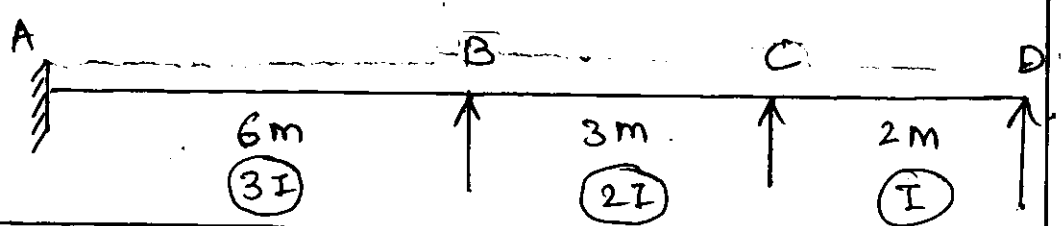
	0.5	0.5		0.5	0.5		Remark
-30	+30	-58.59	+35.156	-30	+30		S.S. mmt zero & Carryover
+30 $\xrightarrow{50\%}$	+15			-15	$\xrightarrow{50\%}$ -30		
0	+45	-58.59	+35.156	45	0		Balancing moment
	+6.795	+6.795	+4.922	+4.922			
0	+51.795	-51.795	+40.078	-40.078	0		

Support Moments.
 $M_A = 51.795 \text{ KN}\cdot\text{m}$
 $M_B = 40.078 \text{ KN}\cdot\text{m}$

2 Marks.

Q. 4(F)

Calculation of Distribution factor



Joint	Member	Stiffness Factor	Total Stiffness	Distribution factor
B	BA	$k_{BA} = \frac{4EI}{L} = \frac{4(3EI)}{6}$ $k_{BA} = 2EI$	$\Sigma k_B = 2 + 2.667 = 4.667EI$	$DF_{BA} = \frac{2EI}{4.667EI}$ $DF_{BA} = 0.428$
	BC	$k_{BC} = \frac{4EI}{L} = \frac{4(2I)}{3}$ $k_{BC} = 2.667EI$		$DF_{BC} = \frac{2.667EI}{4.667EI}$ $DF_{BC} = 0.570$
C	CB	$k_{CB} = \frac{4EI}{L} = \frac{4E(2I)}{3}$ $k_{CB} = 2.667EI$	$\Sigma k_C = 2.667EI + 1.5EI = 4.167EI$	$DF_{CB} = \frac{2.667EI}{4.167EI}$ $DF_{CB} = 0.64$
	CD	$k_{CD} = \frac{3EI}{L} = \frac{3E(I)}{2}$ $k_{CD} = 1.5EI$		$DF_{CD} = \frac{1.5EI}{4.167EI}$ $DF_{CD} = 0.36$

1 Mark

1 Mark

1 Mark

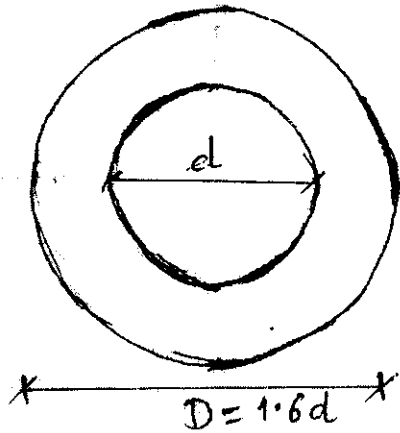
1 Mark

Q.5 Attempt Any TWO

2x8

(16)

(a)



where $h = 32\text{ m}$

$$\rho_m = 18\text{ kN/m}^3$$

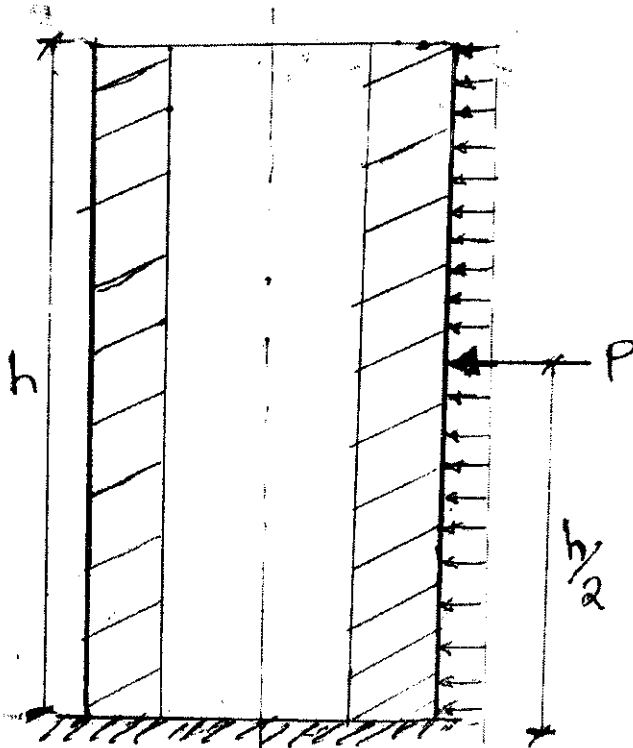
$$C = 0.6$$

$$p_d = 1.75\text{ kN/m}^2$$

$$f_{\text{max}} = ?$$

$$f_{\text{min}} = ?$$

No tension at base.



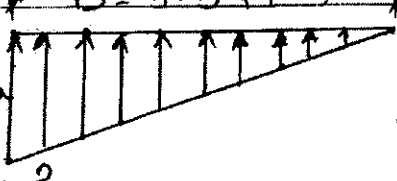
Direct stress due to self weight (f_d):-

$$f_d = \frac{\rho_m A h}{A} = 18 \times 32 = 576\text{ kN/m}^2$$

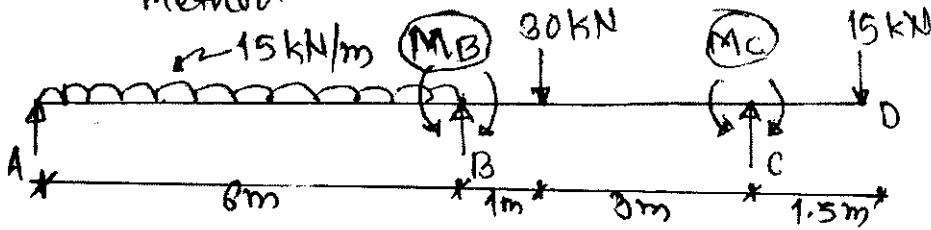
(4M)

Bending stress due to wind load (f_b):-

Bending Moment due to wind force (M)

Question and Model Answers	Marks
$M = P \times h/2$	
$M = (0.6 \times 1.75) (D \times 32) (32/2)$	(2M)
$M = 860.16 d \text{ kN}\cdot\text{m} \quad \dots \because D = 1.6d$	
<p>Moment of Inertia (I) about bending axis</p>	
$I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi((1.6d)^4 - d^4)}{64}$	(1M)
$I = 0.273 d^4 \text{ m}^4$	
<p>Distance of extreme fibre from N.A.</p>	
$y = \frac{D}{2} = \frac{1.6d}{2} = 0.8D$	(1M)
$\therefore f_b = \frac{M}{I} \cdot y = \frac{860.16 d}{0.273 d^4} \times 0.8 d$	(1M)
$\therefore f_b = 2520.62/d^2 \text{ KN/m}^2$	(1M)
<p>To avoid tension at base of chimney,</p>	
$f_d = f_b$	
$\therefore 576 = \frac{2520.62}{d^2}$	
<p>∴ Internal Diameter = $d = 2.092 \text{ m}$ & External Diameter = $D = 3.347 \text{ m}$ (1M)</p>	
<p>Extreme fibre stress at the base of chimney</p>	
<p>$f_d + f_b = f_{max}$ $2(576) = 1152 \text{ KN/m}^2$</p>  <p>$f_{min} = f_d - f_b$ $f_{min} = 0 \text{ KN/m}^2$ at the windward side</p>	(1M)
<p>∴ the leeward side.</p>	

Q5 (B) Draw BMD from Moment Distribution Method.



Distribution factors At jt. B. - $\frac{DF}{\sum DF}$

Jt. B	{	BA	Relative stiffness = $\frac{3EI}{6}$	= $\frac{0.4}{\underline{0.4}}$
		BC	= $\frac{3EI}{4}$	= $\frac{0.6}{\underline{0.6}}$
			Total stiffness = $1.25EI$	$\sum = 1.0$

(1M)

Fixed End Moments (FEM)

$$M_{AB} = \frac{15 \times (6)^2}{12} = -45 \text{ kN}\cdot\text{m} \text{ (anticlockwise)} \quad (\frac{1}{2}M)$$

$$M_{BA} = \frac{15 \times (6)^2}{12} = +45 \text{ kN}\cdot\text{m} \text{ (clockwise)} \quad (\frac{1}{2}M)$$

$$M_{BC} = \frac{30 \times 1 \times (3)^2}{(4)^2} = -16.875 \text{ kN}\cdot\text{m} \text{ (anticlockwise)} \quad (\frac{1}{2}M)$$

$$M_{CB} = \frac{30 \times (1)^2 \times 3}{(4)^2} = +7.5 \text{ kN}\cdot\text{m} \text{ (clockwise)} \quad (\frac{1}{2}M)$$

$$M_{CD} = 15 \times 1.5 = -22.5 \text{ kN}\cdot\text{m} \text{ (anticlockwise)} \quad (\frac{1}{2}M)$$

Sagging BM. for (s/s for every span)

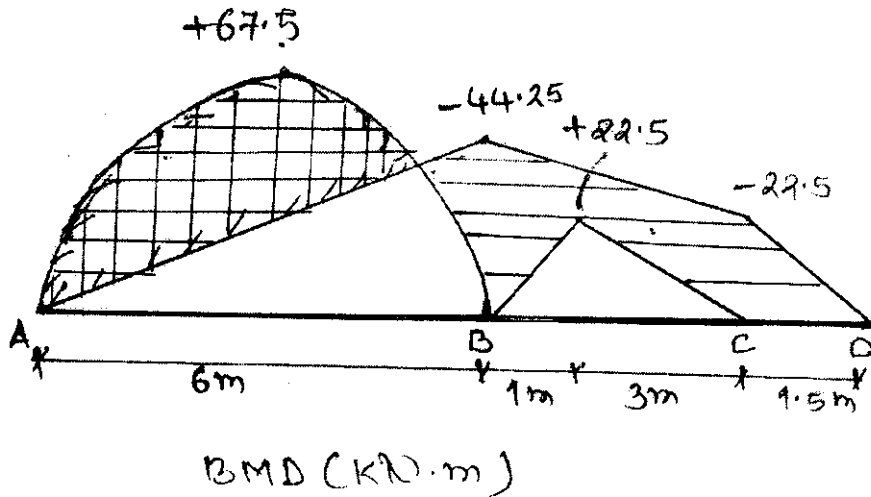
$$M_{AB}^+ = \frac{15 \times (6)^2}{8} = +67.5 \text{ kN}\cdot\text{m} \quad (\frac{1}{2}M)$$

$$M_{BC}^+ = \frac{30 \times 1 \times 9}{4} = +22.5 \text{ kN}\cdot\text{m}$$

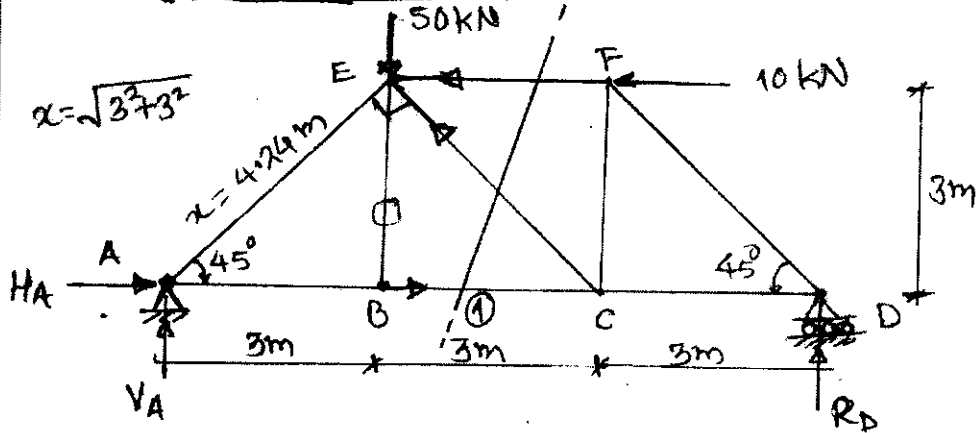
Subject Code = 17422

Question and Model Answers						Marks
Moment Distribution Table						
Joint	A	B		C	D	
D.F.	-	0.4	0.6	1.0	-	
FEM	-45	+45	-16.875	+7.5	-22.5	
Release A & Balance C	+45	+22.5	+7.50	+15.0		
Initial Moment	0	+67.50	-9.375	+22.5	-22.5	
Distribute 'B'		-23.25	-34.875			
Final Moment	0	+44.25	-44.25	+22.5	-22.5	(2M)

Release 'A': Because of s/s end.
 Balance 'C': Because of overhang



Q.5 (c) Method of section



Calculate Support Reaction using condition of Equilibrium for whole truss.

$$\sum F_x = 0 \quad \therefore \underline{H_A = 10 \text{ kN}}$$

(1/2 M)

$$\sum M @ A = 0 = +(50 \times 3) - (10 \times 3) - (R_D \times 9)$$

(1/2 M)

$$\therefore \underline{R_D = 13.33 \text{ kN}}$$

$$\sum F_y = 0 \quad \therefore \underline{V_A = 50 - 13.33 = 36.67 \text{ kN}}$$

(1/2 M)

Taking section (1)-(1) as shown in fig, & using conditions of Equilibrium as follows for sections,

Taking moment @ E

$$\sum M_E = 0 = +(36.67 \times 3) - (10 \times 3) - (F_{BC} \times 3)$$

$$\therefore \boxed{F_{BC} = +26.67 \text{ kN}}$$

Assumed direction is OK \therefore Tension

(1/2 M)

Taking moment @ C

$$\sum M_C = 0 = +(36.67 \times 6) - (50 \times 3) - (F_{EF} \times 3)$$

$\therefore \boxed{F_{EF} = +23.34 \text{ kN}}$ \therefore Assumed direction is ok. \therefore Compressive

(1M)
(2)

Taking moment @ A

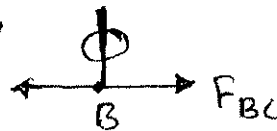
$\sum M_A = +(50 \times 3) - (F_{EC} \times 4.24) - (23.34 \times 3) = 0$

$\therefore \boxed{F_{EC} = +18.86 \text{ kN}}$ \therefore Assumed direction is ok. \therefore Compressive

(1M)

\therefore At joint B for equilibrium,

$\boxed{F_{BE} = 0}$



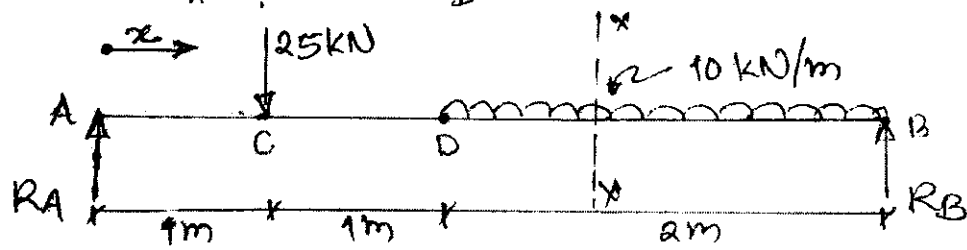
(1/2M)

Result in Tabulate form

SR No.	MEMBER	MAGNITUDE (kN)	NATURE
1.	BC	26.67	Tension
2.	BE	0	—
3.	EF	23.34	Compressive
4.	EC	18.86	Compressive

(2M)

subject code = 17422

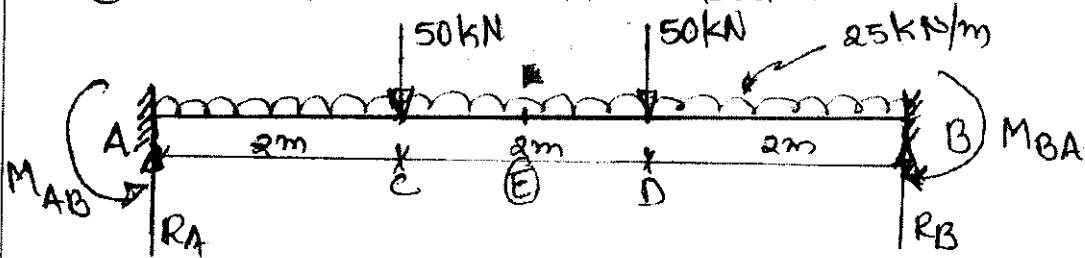
Question and Model Answers	Marks
<p>Q.6 Attempt Any TWO 2x8</p>	(16)
<p>Q.6. (a) Slope & deflection by Macaulay's Method.</p>	
<p style="text-align: center;">$\theta_A = ?$ $Y_B = ?$ $EI = 40,000 \text{ kN}\cdot\text{m}^2$</p>	
	
<p>Using conditions of Equilibrium, -</p>	
$\sum M @ A = 0 = +(25 \times 1) + (10 \times 2 \times \frac{2+2}{2}) - (R_B \times 4)$	
$\therefore \boxed{R_B = 21.25 \text{ kN}}$	(1M)
$\sum F_y = 0 = -25 - (10 \times 2) + (21.25) + R_A$	
$\therefore \boxed{R_A = 23.75 \text{ kN}}$	(1M)
<p>For Macaulay's Method, section x-x as shown in fig with origin @ A.</p>	
$M_{xx} = 23.75x - (25)(x-1) - \frac{10(x-2)^2}{2}$	(1M)
<p>Using Macaulay's Method,</p>	
$EI \frac{d^2y}{dx^2} = M_{xx} = 23.75x - 25(x-1) - 5(x-2)^2$	(1M)
<p>Integrating on both side,</p>	
$EI \frac{dy}{dx} = \frac{23.75x^2}{2} + C_1 - \frac{25(x-1)^2}{2} - \frac{5(x-2)^3}{3}$	(1M)
<p>Integrating on both side, -</p>	
$EI y = \frac{23.75x^3}{6} + C_1x + C_2 - \frac{25(x-1)^3}{6} - \frac{5(x-2)^4}{12}$	(1M)

Subject code: 17422

Question and Model Answers	Marks
<p>using boundary condition, -</p> <p>i) at $x=0$, $y=0$</p> $EI y = 0 = C_2 = 0$ <p>ii) at $x=4$, $y=0$</p> $0 = EI y = \frac{23.75(4)^3}{6} + C_1(4) + 0 \left - \frac{25(4-1)^3}{6} \right - \frac{5(4-2)^4}{12}$ $\therefore C_1 = -33.54$ <p>for slope at end span @ $x=0$</p> $EI \frac{dy}{dx} = \frac{23.75x^2}{2} - 33.54 \left - \frac{25(x-1)^2}{2} \right - \frac{5(x-2)^3}{3}$ $EI \theta_A = -33.54$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $\theta_A = \frac{-33.54}{40,000} = 0.8385 \times 10^{-3} \text{ rad}$ </div> <p>For displacement at mid span @ $x=2m$</p> $EI y = \frac{23.75x^3}{6} - 33.54(x) \left - \frac{25(x-1)^3}{6} \right - \frac{5(x-2)^4}{12}$ $EI y_D = \frac{23.75(2)^3}{6} - 33.54(2) \left - \frac{25(2-1)^3}{6} \right - \frac{5(2-2)^4}{12}$ $\therefore EI y_D = -39.58$ $y_D = \frac{-39.58}{40,000} = -0.9895 \times 10^{-3} \text{ m}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $y_D = -0.9895 \text{ mm}$ </div> <p>-ve sign indicates vertically downwards displacement</p>	<p>(1)</p> <p>(1)</p>

Question and Model Answers	Marks
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Q.6 (b) SFD & BMD of fixed beam



As symmetrical loading,

$$R_A = R_B = \frac{50 + 50 + (25 \times 6)}{2} = 125 \text{ kN} \quad (1M)$$

$$\& M_{AB} = M_{BA} = \frac{50(2)(4)^2}{(6)^2} + \frac{50(4)(2)^2}{(6)^2} + \frac{25(6)^2}{8} \quad (1M)$$

$$\therefore M_{AB} = -141.66 \text{ kN}\cdot\text{m} \text{ (anticlockwise)} \quad (1M)$$

$$M_{BA} = +141.66 \text{ kN}\cdot\text{m} \text{ (clockwise)}$$

S.F. calculation

$$\text{S.F. @ A} = 125 \text{ kN}$$

$$\text{S.F. @ C} = 125 - 25 \times 2 \times \frac{2}{2} = 75 \text{ kN} \quad (1M)$$

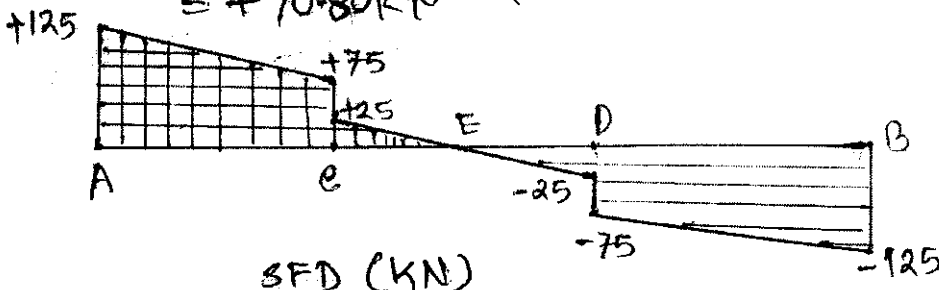
$$\text{S.F. @ C with 50 kN} = +75 - 50 = +25 \text{ kN}$$

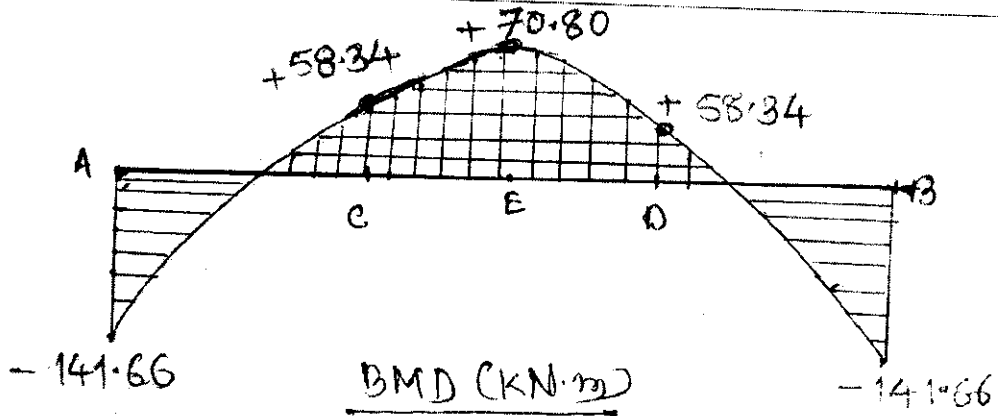
BM. calculation

$$\text{BM @ A} = -141.66 \text{ kN}\cdot\text{m}$$

$$\text{BM @ C} = -141.66 + (125 \times 2) - \left(25 \times 2 \times \frac{2}{2}\right) = +58.34 \text{ kN}\cdot\text{m} \quad (1M)$$

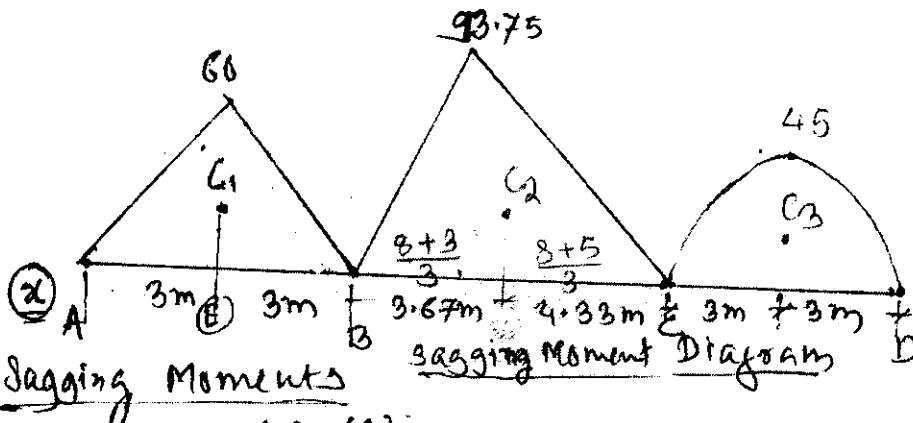
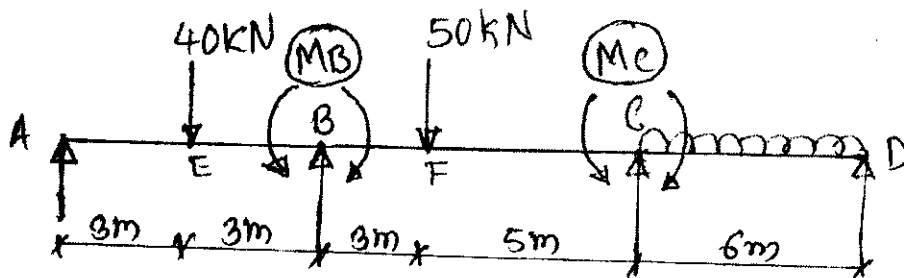
$$\text{BM @ E} = -141.66 + (125 \times 3) - \left(25 \times 3 \times \frac{3}{2}\right) - (50 \times 1) = +70.80 \text{ kN}\cdot\text{m}$$





(2M)

Q.6 (c) Support Reactions & Moments by Three moment theorem.



(1M)

Span AB = $\frac{40 \times 6^2}{4} = 60 \text{ kN.m}$

(1/2M)

Span BC = $\frac{50 \times 3 \times 5}{8} = 93.75 \text{ kN.m}$

(1/2M)

Subject code: 17422

Question and Model Answers	Marks
$\text{span } CD = \frac{16 \times 6^2}{8} = 45 \text{ KN}\cdot\text{m}$	(1/2 M)
Area of Moment diagram:-	(1/2 M)
for span AB = $a_1 = \frac{1}{2} \times 6 \times 60 = 180$	(1/2 M)
for span BC = $a_2 = \frac{1}{2} \times 8 \times 93.75 = 375$	(1/2 M)
for span CD = $a_3 = \frac{1}{2} \times 6 \times 45 = 135$	(1/2 M)
Using Three Moment Theorem for	
Pair ABC	
$(M_A \times 6) + 2M_B(6+8) + (M_C \times 8) = \frac{-6(180 \times 3)}{6}$	
As $M_A = 0$, (s/s at end)	
$28M_B + 8M_C = -1757.82 \quad \dots \textcircled{1}$	(1 M)
Pair B-C-D	
$(M_B \times 8) + 2M_C(8+6) + (M_D \times 6) = \frac{-6(375)(3.67)}{8}$	
As $M_D = 0$, (s/s at end)	
$8M_B + 28M_C = -1572.19 \quad \dots \textcircled{2}$	(1 M)
Solving eqn $\textcircled{1}$ & $\textcircled{2}$	
$M_B = -50.89 \text{ KN}\cdot\text{m} \quad \text{(Hogging)}$	(1/2 M)
$M_C = -41.61 \text{ KN}\cdot\text{m} \quad \text{(Hogging)}$	(1/2 M)

Subject code : 17422

Question and Model Answers

Marks

for Support Reaction

① Take section at B & moment at left side of B

$$(R_A \times 6) - (40 \times 3) = -50.89$$

$$\therefore \boxed{R_A = +11.52 \text{ kN}}$$

② Take section at C & moment at right side of C

$$(R_D \times 6) - (10 \times 6 \times \frac{6}{2}) = -41.61$$

$$\therefore \boxed{R_D = +23.07 \text{ kN}}$$

(1/2M)

③ Take section at C & moment at left side of C

$$(11.52 \times 14) - (40 \times 11) + (R_B \times 8) - (50 \times 5) = -41.61$$

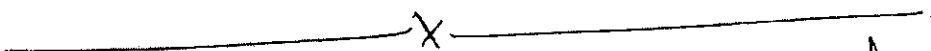
$$\boxed{R_B = +60.89 \text{ kN}}$$

④ Using $\sum F_y = 0$ for overall beam,

$$R_A + R_B + R_C + R_D = 40 + 50 + (10 \times 6)$$

$$\therefore \boxed{R_C = 54.52 \text{ kN}}$$

(1/2M)



Ans
24/11/16