## 23124

## 3 Hours / 70 Marks

$\square$
Instructions - (1) All Questions are Compulsory.
(2) Answer each next main Question on a new page.
(3) Illustrate your answers with neat sketches wherever necessary.
(4) Figures to the right indicate full marks.
(5) Assume suitable data, if necessary.
(6) Use of Non-programmable Electronic Pocket Calculator is permissible.
(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

## Marks

1. Solve any FIVE of the following: $\mathbf{1 0}$
a) If $f(x)=2^{x}-\log _{2}{ }^{x}$ then find $f(2)$.
b) Test whether the function is even or odd, if $f(x)=3 x^{4}-2 x^{2}+\cos x$.
c) Find $\frac{d y}{d x}$ if $y=e^{x} \cdot \cot x$.
d) Evaluate $\int \frac{1}{3 x-2} d x$.
e) Evaluate $\int \frac{2+3 \sin x}{\cos ^{2} x} d x$.
f) Find the area bounded by the curve $y=3 x^{2}, x$-axis and the ordinates $x=1$ and $x=3$.
g) Express into polar form $\mathrm{z}=\frac{1}{2}+\frac{\sqrt{3}}{2} \mathrm{i}$.
2. Solve any THREE of the following:
a) If $x^{2}+y^{2}+x y-y=0$ find $\frac{d y}{d x}$ at $(1,2)$.
b) Find $\frac{d y}{d x}$ if $y=\tan ^{-1}\left[\frac{2 x}{1+35 x^{2}}\right]$.
c) Find the radius of curvature of the curve $y^{2}=4 x$ at point ( $2, \sqrt{2}$ ).
d) An electric cable cost ₹ C per kilometer and $\mathrm{C}=\frac{100}{x}+625 x$ where $x$ is the area of cross-section. Find the value of $x$ for which cost is minimum. What is minimum cost?
3. Solve any THREE of the following:
a) Find the equation of tangent to the circle.
$x^{2}+y^{2}-8 x+4 y+12=0$ at point $(2,0)$.
b) If $x=a \sin \theta$ and $y=a(1+\cos \theta)$, find $\frac{d y}{d x}$.
c) If $e^{x}=x^{y}$, prove that $\frac{d y}{d x}=\frac{\log x-1}{(\log x)^{2}}$.
d) Evaluate $\int \frac{1}{x\left[16+\left(\log _{e}\right)^{2}\right]} d x$.
4. Solve any THREE of the following:
a) Evaluate $\int \frac{1}{2 x^{2}+3 x+1} d x$.
b) Evaluate $\int \frac{1}{4+5 \sin 2 x} d x$.
c) Evaluate $\int x \cdot \log (1+x) d x$.
d) Evaluate $\int \frac{e^{x}}{\left(e^{x}-1\right)\left(e^{x}+1\right)} d x$.
e) Evaluate $\int_{1}^{3} \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}} d x$.
5. Solve any TWO of the following: 12
a) Find the area bounded by two parabolas $y^{2}=2 x$ and $x^{2}=2 y$.
b) i) Find order and degree of differential equation.

$$
\frac{d^{2} y}{d x^{2}}=\left[y+\frac{d y}{d x}\right]^{3 / 2}
$$

ii) Solve $\frac{d y}{d x}=e^{x} \cdot e^{-y}+x e^{-y}$
c) A resistance of $100 \Omega$ and inductance of 0.1 henries are connected in series with a battery of 20 volts. Find the current in the circuit at any instant, if the relation between
$L, R, E$ is $L \frac{d i}{d t}+R i=E$.
6. Solve any TWO of the following: 12
a) i) Express $\frac{(3-i)^{2}}{2-i}$ in ' $x+i y^{\prime}$ form.
ii) Find $L\left\{e^{-t} \cos 2 t\right\}$
b) Find $L^{-1}\left\{\frac{4 s+5}{(s+2)(s-1)^{2}}\right\}$
c) Solve given differential equation by using Laplace transform.

$$
\frac{d y}{d t}+3 y=2+e^{-t}, \text { where } y(0)=1
$$

