23124

3 Hours / 70 Marks Seat No. | | | | | | | | |

Instructions : (1) All Questions are *compulsory.*

(2) Answer each Section on same / separate answer sheet.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

Marks

1. Attempt any FIVE of the following : **10**

(a) If $f(x, y) = 3x + 4xy$ find $\frac{\partial f}{\partial x}$.

(b) If $f(x, y) = x^2y + \sin x + \cos y$ find $\frac{\partial^2 f}{\partial x \partial y}$.

(c) Find the Eigen values of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(d) Find rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.

(e) Find the value of 'P', if the vectors $\bar{a} = p\bar{i} + 5\bar{j} + \bar{k}$ & $\bar{b} = 2\bar{i} - \bar{j} + 3\bar{k}$ are equal.

(f) Find the projection of $\bar{a} = 2\bar{i} + \bar{j} + \bar{k}$ on $\bar{b} = \bar{i} + 3\bar{j} + \bar{k}$.

(g) Construct forward difference table for the following data :

x	1	2	3	4	5
$f(x)$	4	13	34	73	136

2. Attempt any THREE of the following : **12**

(a) Examine $f(x, y) = xy$ subject to the constraint $g(x, y) = 4x^2 + y^2 = 8$ for maximum and minimum value.

(b) Examine the following linear system of equation for consistency and solve if consistent :

$$4x - 2y + 6z = 8$$

$$x + y - 3z = -1$$

$$15x - 3y + 9z = 21$$

(c) Reduce the matrix to ECHLON form & hence find its rank.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}$$

(d) Find the angle between the vectors $\bar{a} = 2\bar{i} + 2\bar{j} + \bar{k}$ & $\bar{b} = 3\bar{i} + 6\bar{j} + 2\bar{k}$.

3. Attempt any THREE of the following : **12**

(a) If $f(x, y) = x^3 + y^3 + 6xy$ then find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ & $\frac{\partial^2 f}{\partial y \partial x}$.

(b) Find inverse of following matrix by elementary transformation :

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

(c) Determine the value of λ for which the system of linear equation $3x + 2y + 4z = 3$, $x + y + z = \lambda$, $5x + 4y + 6z = 15$ are consistent, also find its solution.

(d) Find the local minima of function $f(x, y) = 2x^2 + 2xy + 2y^2 - 6x$.

4. Attempt any THREE of the following : **12**

(a) Show that the equations $2x + 6y = -11$, $6x + 20y - 6z = -3$, $6y - 18z = -1$ are not consistent.

(b) Find a vector of magnitude $\sqrt{7}$ units & perpendicular to the vectors $\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$ & $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$.

(c) Find the Scalar product of the vector $\bar{a} = \bar{i} + \bar{j} + \bar{k}$ & $\bar{b} = 2\bar{i} + 4\bar{j} - 5\bar{k}$ & $\bar{c} = 2\bar{i} + 2\bar{j} + 3\bar{k}$.

(d) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 14 & -10 \\ 5 & -1 \end{bmatrix}$.

(e) If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ & $\bar{c} = \bar{j} + \bar{k}$ find vector $\bar{a} \times (\bar{b} \times \bar{c})$.

5. Attempt any TWO of the following : **12**

(a) Find $\frac{dy}{dx}$ at $x = 0$ using suitable interpolation table :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(b) Find f(12) using Newton's forward difference interpolation table :

x	10	15	20
f(x)	14	18	28

(c) Solve the following linear programming problem graphically to find optimal solution :

Maximize $z = 5x + 3y$

Subject to $3x + 5y \leq 15$

$5x + 2y \leq 10$

$x \geq 0, y \geq 0$

6. Attempt any TWO of the following : **12**

(a) Given the squares of integers in the following data. Find the value of $(13)^2$ using extrapolation.

x	3	5	7	9
y	9	25	49	81

(b) (i) Evaluate $\int_2^7 \frac{1}{x}\,dx$ using trapezoidal rule and by dividing the interval [2, 7] into five equal sub-intervals.

(ii) Evaluate $\int_0^2 \sqrt{x}\,dx$ by using Simpson's one third rule, by dividing the interval (0, 2) into four sub-intervals.

(c) Solve the following linear programming problem using Simplex method to find optimal solution :

Maximize $z = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 450$

$2x_1 + x_2 \leq 600$

$x_1, x_2 \geq 0$